MATHEMATICS CONTENT MODULES

Module 5: Recognizing and Generating Equivalent Expressions

Session 0: Setting the Stage for Module 5
Grades 6-9

The goal of the initiative

Improve the quality of instruction in math classrooms by developing deep mathematical content knowledge of teachers and connecting that content knowledge to the practical implementation of a quality curriculum.

Math Content Module 5
Grades 6-9

2

Group norms

- Understand that those who work, learn.
- Look for solutions, not blame.
- Focus on systems, not people.
- Recognize that everyone has expertise.
- Be honest.
- Challenge ideas.
- Share talk time.
- Mistakes are expected, respected, and inspected.

Module 5 goals

Participants will

- Experience math tasks from EngageNY and other Tier 1 resources related to the focus content of recognizing and generating equivalent expressions.
- Explore coherence between the LSSM related to recognizing and generating equivalent expressions and identify implications for teaching and learning.
- Investigate the impact of selecting student responses that highlight specific strategies or misconceptions and sequencing them to be presented in a way that helps students to make connections and deepen their mathematical understanding.
- Use tools for purposeful planning to develop a common understanding of grade-level/course standards to align teaching and learning using EngageNY and other Tier 1 resources.

Math Content Module 5
Grades 6-9

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MATHEMATICS CONTENT MODULES

Module 5: Recognizing and Generating Equivalent Expressions

Session 1: Deepening Mathematical Content Knowledge and Exploring Coherence in the LSSM

Engaging in the mathematics: Recognizing and generating equivalent expressions

Ultimate Invaders Grades 6-9

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Reflection on Ultimate Invaders

How might an activity like this help students make sense of equivalent expressions?

Properties of operation

- Identify the standards at your grade level that either introduce students to properties of operation or use them.
- Make a poster summarizing the work at your grade level and be prepared to present to the whole group.

Recognizing and generating equivalent expressions across the grades

M5_P-02 Recognizing and generating equivalent expressions

Fill in the chart summarizing the key understandings at each grade level.

Key Understandings could include:

- Standards
- Consolidated summary of content in standards
- If a skill or concept is introduced, extended, or finalized

	Key Understandings
Algebra I	
8th grade	
7th grade	
6th grade	

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Reflection

Record your individual reflection on the following questions on a sticky note.

- What did you learn about the vertical progression of recognizing and generating equivalent expressions across the grades?
- What actions will you take in your classroom instruction based on what you learned?

Math Content Module 5, Session 1
Grades 6-9

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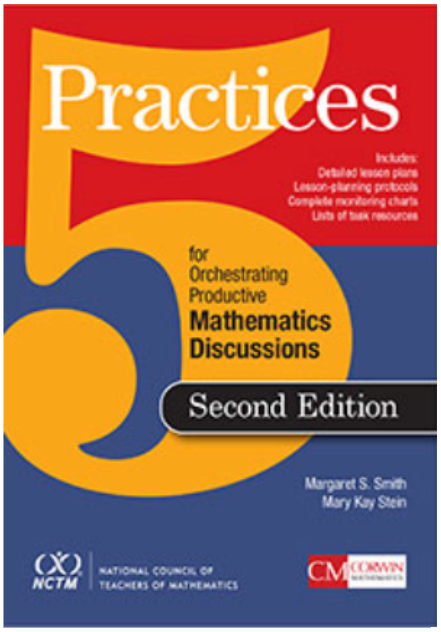
MATHEMATICS CONTENT MODULES

Module 5: Recognizing and Generating Equivalent Expressions

Session 2: Instructional Strategies to Improve Curriculum Implementation

Grades 6-9

5 Practices for Orchestrating Productive Mathematics Discussions



— Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions.* Reston, VA: National Council of Teachers of Mathematics. Available at http://www.nctm.org/Store/Products/5--Practices-for-Orchestrating-Productive-Mathematics-Discussions

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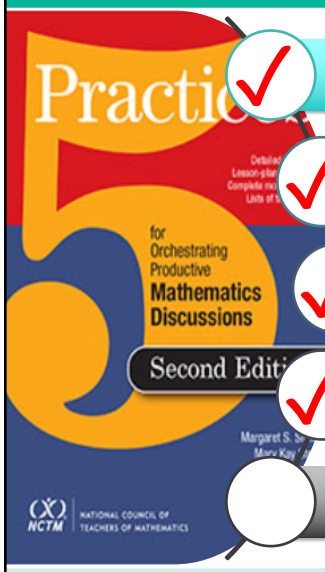
5 Practices for Orchestrating Productive Mathematics Discussions



- *Anticipating* student responses prior to the lesson
- *Monitoring* students' work on and engagement with the tasks
- *Selecting* particular students to present their mathematical work
- *Sequencing* students' responses in a specific order for discourse
- *Connecting* different students' responses, and connecting the responses to key mathematical ideas

Orchestrating productive mathematics discussions

Anticipating student responses prior to the lesson

What should you consider?	How is this supported?
• The strategies that students might use to approach or solve a challenging mathematical task • How to respond to what students product • Which strategies are most useful in addressing the mathematics to be learned	• *Solving the problem in as many ways as possible* • Solving the problem with other teachers • Drawing on relevant research when possible • Document students responses year to year

— Adapted from Mary Kay Stein and Margaret Schwan Smith. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions.* |Reston, VA: National Council of Teacher of Mathematics.

Orchestrating productive mathematics discussions

Monitoring students' work on, and engagement with, the task

What does this involve?	How is this supported?
• Circulating while students work, watching and listening • Recording interpretations, strategies, and points of confusion • Asking probing questions to get students back "on track" or to advance their understanding	• *Anticipating* student responses beforehand • Using a recording tool • Observing students' actual responses during independent work

— Adapted from Mary Kay Stein and Margaret Schwan Smith. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions.* |Reston, VA: National Council of Teacher of Mathematics.

Math Content Module 5, Session 2
Grades 6-9

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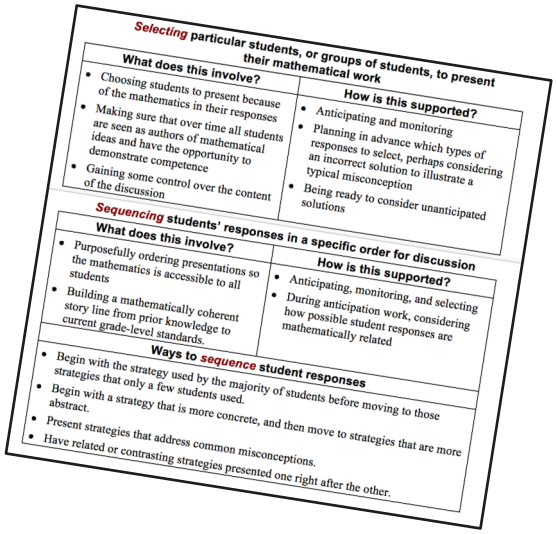
Orchestrating productive discussions

1 – Selecting

2 – Sequencing

Read your assigned practice.

- Make note of important takeaways.
- Be prepared to share.



Selecting particular students, or groups of students, to present their mathematical work

What does this involve?	How is this supported?
• Choosing students to present because of the mathematics in their responses • Making sure that over time all students are seen as authors of mathematical ideas and have the opportunity to demonstrate competence • Gaining some control over the content of the discussion	• Anticipating and monitoring • Planning in advance which types of responses to select, perhaps considering an incorrect solution to illustrate a typical misconception • Being ready to consider unanticipated solutions

Sequencing students' responses in a specific order for discussion

What does this involve?	How is this supported?
• Purposefully ordering presentations so the mathematics is accessible to all students • Building a mathematically coherent story line from prior knowledge to current grade-level standards.	• Anticipating, monitoring, and selecting • During anticipation work, considering how possible student responses are mathematically related

Ways to _sequence_ student responses

- Begin with the strategy used by the majority of students before moving to those strategies that only a few students used.
- Begin with a strategy that is more concrete, and then move to strategies that are more abstract.
- Present strategies that address common misconceptions.
- Have related or contrasting strategies presented one right after the other.

Math Content Module 5, Session 2
Grades 6-9

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Orchestrating productive discussions

Selecting **particular students, or groups of students, to present their mathematical work**

What does this involve?	How is this supported?
• Choosing students to present because of the mathematics in their responses • Making sure that over time, all students are seen as authors of mathematical ideas and have the opportunity to demonstrate competence • Gaining some control over the content of the discussion	• *Anticipating* and *monitoring* • Planning in advance which types of responses to select, perhaps considering an incorrect solution to illustrate a typical misconception • Being ready to consider unanticipated solutions

— Adapted Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions.* Reston, VA: National Council of Teachers of Mathematics.

Math Content Module 5, Session 2
Grades 6-9
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Orchestrating productive discussions

Sequencing **students' responses in a specific order for discussion**

What does this involve?	How is this supported?
• Purposefully ordering presentations so the mathematics is accessible to all students • Building a mathematically coherent story line from prior knowledge to current grade-level standards	• *Anticipating*, *monitoring*, and *selecting* • During anticipation of work, considering how possible student responses are mathematically related

— Adapted from Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions.* Reston, VA: National Council of Teachers of Mathematics.

Math Content Module 5, Session 2
Grades 6-9
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Orchestrating productive discussions

Ways to ***Sequence***

- Begin with the strategy used by the majority of students before moving to those strategies that only a few students used.
- Begin with a strategy that is more concrete and then move to strategies that are more abstract.
- Present strategies that address common misconceptions.
- Have related or contrasting strategies presented one right after the other.

— Adapted from Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions.* Reston, VA: National Council of Teachers of Mathematics.

Math Content Module 5, Session 2
Grades 6-9

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Recall the Ultimate Invaders problem from Session 1.

Learning Goal:
Students will understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities are related.

Credit: Agile Mind, Inc.

Turn and Talk:

- Discuss at least 2 student responses you would anticipate for the Ultimate Invaders problem.

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Orchestrating productive discussions

With your group, analyze the student work samples for strategies and misconceptions. Then...

- ***Select*** student work that would best represent the strategies noted in the ***Anticipating*** section or that would help build understanding of the math.
  - In the **Planning for Mathematical Discourse** tool, make note of which students used the anticipated strategies and describe the strategy used.
- ***Sequence*** the selected student work in the order that your group determines would best help the students make sense of the math and make note of the sequence.

Math Content Module 5, Session 2
Grades 6-9
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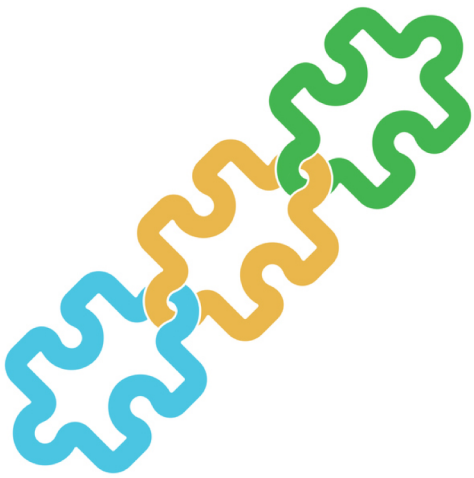
Orchestrating productive discussions



CONNECT your knowledge of the the 5 Practices to what you already know?

EXTEND your thinking in new directions?

CHALLENGE previous thoughts or ideas?

Math Content Module 5, Session 2
Grades 6-9
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MATHEMATICS CONTENT MODULES

Module 5: Recognizing and Generating Equivalent Expressions

Session 3: Key Shifts in Action through an EngageNY Lesson

Grades 6-9

EngageNY module overview

- Overview
- Focus Standards
- Foundational Standards

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Evidence of student engagement in the key shifts

Use the handout to:

- Check off student behaviors/actions that you observed.
- Record any observations you made that serve as evidence.

Looking for Evidence of Student Engagement in the Key Shifts

Focus	Evidence
☐ The learning goal(s) of the lesson supports grade level standard(s).	
Coherence	
☐ The lesson intentionally relates new concepts to students' prior skills and knowledge. ☐ Students set the foundation for future learning. ☐ Students access prior learning from major work in the grade in order to support new learning.	
Rigor	
Conceptual Understanding ☐ Students access concepts and ideas from a variety of perspectives. ☐ Students explain mathematical ideas behind a particular concept in a variety of ways. ☐ Students use examples and counterexamples to make and support conjectures applied to one problem to multiple situations. ☐ Students create and use a variety of models to analyze relationships. ☐ Students make use of patterns and structure to compose and decompose numbers, shapes, expressions, and equations. **Procedural Skills and Fluency** ☐ Students select tools (e.g. physical objects, manipulatives, drawings, diagrams, algorithms, or strategies) that are relevant and useful for the task or problem. ☐ Students communicate thinking using appropriate vocabulary, symbols and/or units in precise and accurate ways. ☐ Students look for patterns, generalizations, and shortcuts. ☐ Students are flexible in their use of procedures and skills to solve problems. **Application** ☐ Students decontextualize and contextualize quantities in problem situations. ☐ Students plan and choose a solution pathway when applying their mathematical knowledge to different situations.	

Note: To help educators look for evidence of grade-level-appropriate student engagement in mathematical tasks, these narrative descriptors are adapted from Illustrative Mathematics. (2014, February 12). *Standards for Mathematical Practice: Commentary and Elaborations for K–5 and 6–8*. Tucson, AZ. Available at http://commoncoretools.me/2014/02/12/k-5-elaborations-of-the-practice-standards

Math Content Module 5, Session 3
Grades 6-9

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Learning expectations

The student will:

- Use visual models including tape diagrams and rectangular arrays to write products as sums and sums as products.
- Recognize that rewriting an expression in a different form can reveal how quantities are related.
- Use examples and counterexamples to communicate thinking using appropriate vocabulary, symbols, and/or units.

Math Content Module 5, Session 3
Grades 6–9

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Engaging in the math

Solve this problem using a tape diagram.

A sum of money was shared between George and Benjamin in a ratio of 3:4. If the sum of money was $56.00, how much did George get?

Engaging in the math

Represent 3 + 2 using a tape diagram.

Represent x + 2 using a tape diagram.

Engaging in the math

Draw a rectangular array for 3 (3 + 2)

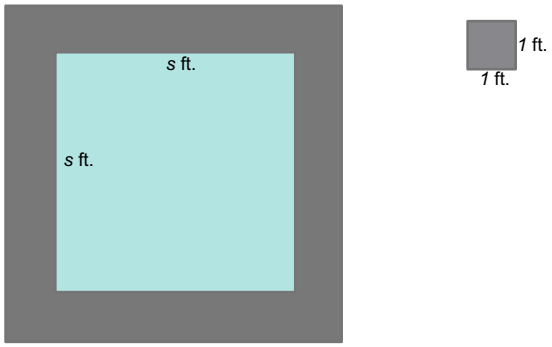
How would this change if the original problem was 3 (x + 2)?

Engaging in the math

Create 3 different representations for the following expression:
3(x + y)

Engaging in the math

A square fountain with side length *s* feet is bordered by a single row of square tiles as shown.



Express the total number of tiles needed in terms of *s* three different ways.

Math Content Module 5, Session 3
Grades 6-9

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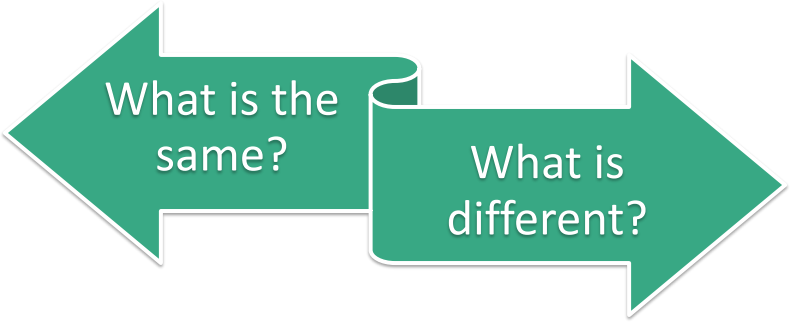
Talking about the math



What did you learn from the problem that you could use in solving other problems?

Math Content Module 5, Session 3
Grades 6-9

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Wrap-up

How did we represent quantities in this lesson? Why do you think we showed varied ways?

Evidence of student engagement in the key shifts

Use the handout to:

- Check off student behaviors/actions that you observed.
- Record any observations you made that serve as evidence.

Looking for Evidence of Student Engagement in the Key Shifts

Focus	Evidence
☐ The learning goal(s) of the lesson supports grade level standard(s).	
Coherence	
☐ The lesson intentionally relates new concepts to students' prior skills and knowledge. ☐ Students set the foundation for future learning. ☐ Students access prior learning from major work in the grade in order to support new learning.	
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Conceptual Understanding ☐ Students access concepts and ideas from a variety of perspectives. ☐ Students explain mathematical ideas behind a particular concept in a variety of ways. ☐ Students use examples and counterexamples to make and support conjectures applied to one problem to multiple situations. ☐ Students create and use a variety of models to analyze relationships. ☐ Students make use of patterns and structure to compose and decompose numbers, shapes, expressions, and equations. **Procedural Skills and Fluency** ☐ Students select tools (e.g. physical objects, manipulatives, drawings, diagrams, algorithms, or strategies) that are relevant and useful for the task or problem. ☐ Students communicate thinking using appropriate vocabulary, symbols and/or units in precise and accurate ways. ☐ Students look for patterns, generalizations, and shortcuts. ☐ Students are flexible in their use of procedures and skills to solve problems. **Application** ☐ Students decontextualize and contextualize quantities in problem situations. ☐ Students plan and choose a solution pathway when applying their mathematical knowledge to different situations.	

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Unpacking the EngageNY lesson: Standards

7.EE.A.1
Apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients.

7.EE.A. 2
Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities are related. ***For example, a + 0.5a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."***

Math Content Module 5, Session 3
Grades 6-9

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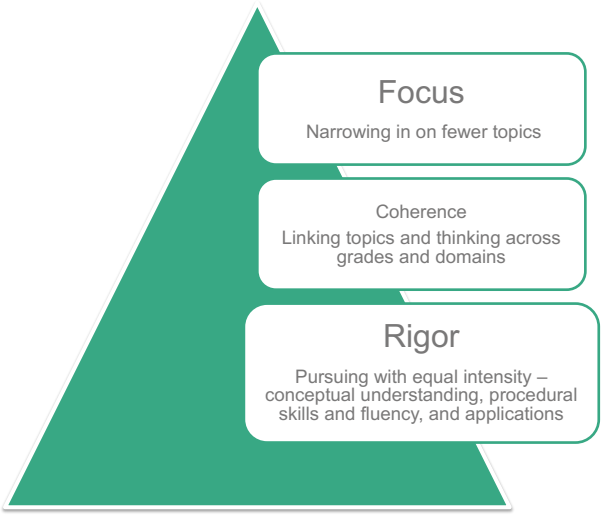
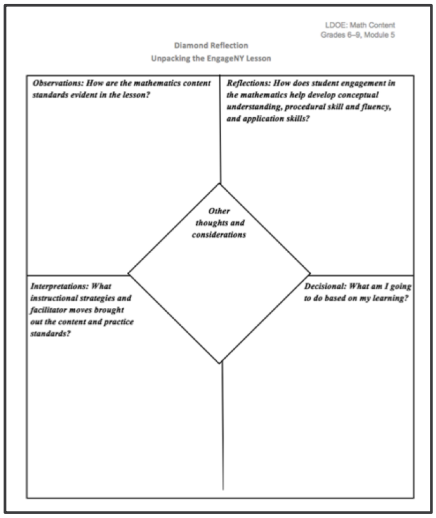
Unpacking the EngageNY lesson: Evidence of key shifts





Math Content Module 5, Session 3
Grades 6-9

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Session reflection

Points to Ponder

- How will unpacking EngageNY lessons for the content standards, SMPs, and key shifts affect your planning and instruction?
- As a result of your learning, identify one *action* you will take to effectively plan for instruction.

Math Content Module 5, Session 3
Grades 6-9

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MATHEMATICS CONTENT MODULES

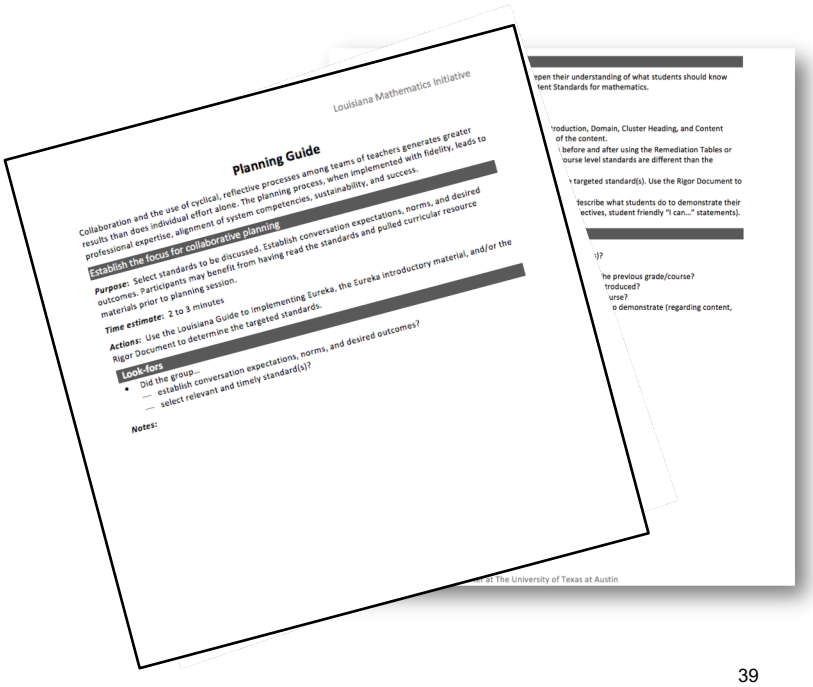
Module 5: Recognizing and Generating Equivalent Expressions

Session 4: Purposeful Planning of the EngageNY Curriculum

Grades 6-9

Tool that promotes effective collaborative planning:

The Planning Guide


Louisiana Mathematics Initiative

Planning Guide

Collaboration and the use of cyclical, reflective processes among teams of teachers generates greater results than does individual effort alone. The planning process, when implemented with fidelity, leads to professional expertise, alignment of system competencies, sustainability, and success.

Establish the focus for collaborative planning

Purpose: Select standards to be discussed. Establish conversation expectations, norms, and desired outcomes. Participants may benefit from having read the standards and pulled curricular resource materials prior to planning session.

Time estimate: 2 to 3 minutes

Actions: Use the Louisiana Guide to Implementing Eureka, the Eureka introductory material, and/or the Rigor Document to determine the targeted standards.

Look-fors

- Did the group…
  - establish conversation expectations, norms, and desired outcomes?
  - select relevant and timely standard(s)?

Notes:

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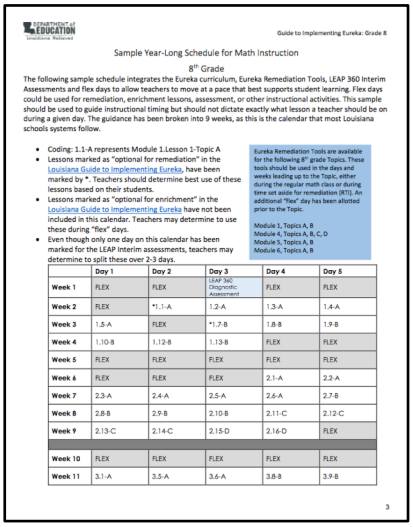
Collaborative planning conversation

Establish the focus standards for collaborative planning:

- Use the *Louisiana Guide to Implementing Eureka* to identify the module and lessons for an upcoming unit.
- Identify the standard(s) related to the upcoming module lessons.


Guide to Implementing Eureka: Grade 8

Sample Year-Long Schedule for Math Instruction

8th Grade

The following sample schedule integrates the Eureka curriculum, Eureka Remediation Tools, LEAP 360 Interim Assessments and flex days to allow teachers to move at a pace that best supports student learning. Flex days could be used for remediation, enrichment lessons, assessment, or other instructional activities. This sample should be used to guide instructional timing but should not dictate exactly what lesson a teacher should be on during a given day. The guidance has been broken into 9 weeks, as this is the calendar that most Louisiana schools systems follow.

- Coding: 1.1-A represents Module 1.Lesson 1-Topic A
- Lessons marked as "optional for remediation" in the Louisiana Guide to Implementing Eureka, have been marked by *. Teachers should determine best use of these lessons based on their students.
- Lessons marked as "optional for enrichment" in the Louisiana Guide to Implementing Eureka have not been included in this calendar. Teachers may determine to use these during "flex" days.
- Even though only one day on this calendar has been marked for the LEAP Interim assessments, teachers may determine to split these over 2-3 days.

Eureka Remediation Tools are available for the following 8th grade Topics. These tools should be used in the days and weeks leading up to the Topic, either during the regular math class or during time set aside for remediation (RTI). An additional "flex" day has been allotted prior to the Topic.

Module 1, Topics A, B
Module 4, Topics A, B, C, D
Module 5, Topics A, B
Module 6, Topics A, B

	Day 1	Day 2	Day 3	Day 4	Day 5
Week 1	FLEX	FLEX	LEAP 360 Diagnostic Assessment	FLEX	FLEX
Week 2	FLEX	*1.1-A	1.2-A	1.3-A	1.4-A
Week 3	1.5-A	FLEX	*1.7-B	1.8-B	1.9-B
Week 4	1.10-B	1.12-B	1.13-B	FLEX	FLEX
Week 5	FLEX	FLEX	FLEX	FLEX	FLEX
Week 6	FLEX	FLEX	FLEX	2.1-A	2.2-A
Week 7	2.3-A	2.4-A	2.5-A	2.6-A	2.7-B
Week 8	2.8-B	2.9-B	2.10-B	2.11-C	2.12-C
Week 9	2.13-C	2.14-C	2.15-D	2.16-D	FLEX
Week 10	FLEX	FLEX	FLEX	FLEX	FLEX
Week 11	3.1-A	3.5-A	3.6-A	3.8-B	3.9-B

3

Math Content Module 5, Session 4
Grades 6-9

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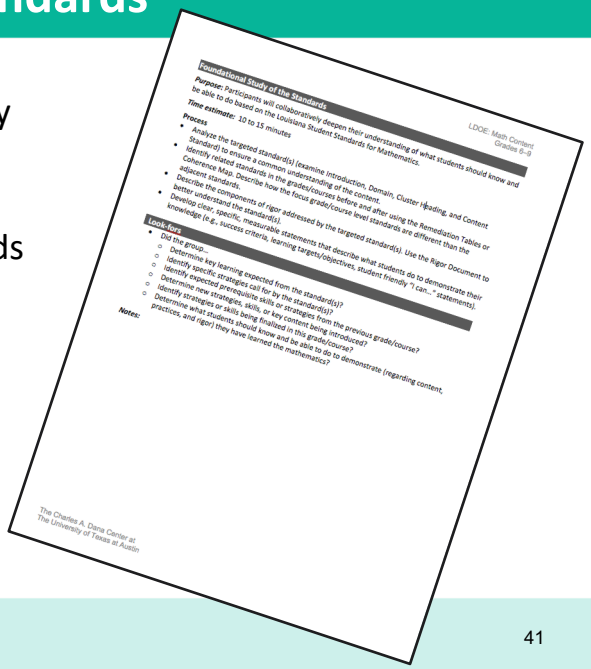
Foundational study of the standards

Purpose: Participants will collaboratively deepen their understanding of what students should know and be able to do based on the Louisiana Student Standards for Mathematics.

Time estimate: 10 to 15 minutes



Math Content Module 5, Session 4
Grades 6-9

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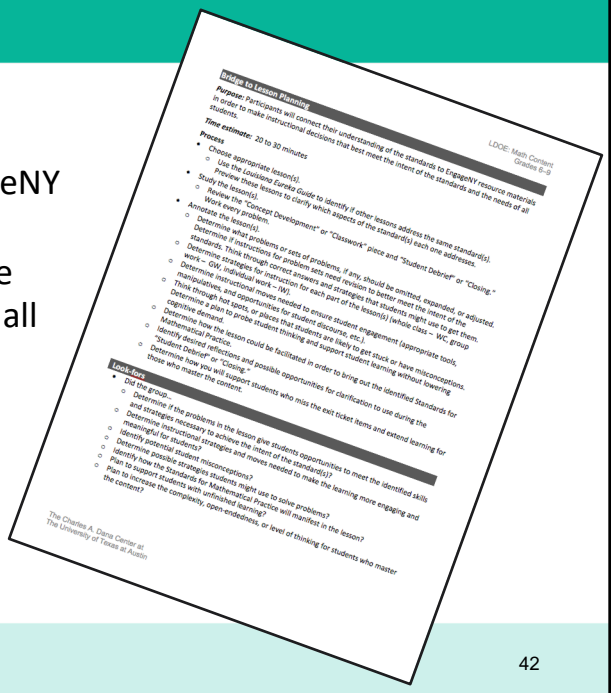
Bridge to lesson planning

Purpose: Participants will connect their understanding of the standards to EngageNY resource materials so they can make instructional decisions that best meet the intent of the standards and the needs of all students.

Time estimate: 20 to 30 minutes



Math Content Module 5, Session 4
Grades 6-9

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Collaborative planning

Work with your grade level group…

- Study your grade level's focus standards using the *Foundational study of the standards*. (15 min)
- Connect your standards conversation to an EngageNY lesson and discuss instructional decisions regarding that lesson using the *Bridge to lesson planning*. (30 min)

Grade 6: 6.NS.B.4, 6.EE.A.3, 6.EE.A.4
Grade 7: 7.EE.A.1, 7.EE.A.2
Grade 8: 8.EE.C.7b
Alg I: A1: A-APR.A.1, A1: A-SSE.B.3a-c, A1: A-REI.A.1, A1: A-REI.B.4a,b

Math Content Module 5, Session 4
Grades 6-9

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Using an Engage NY lesson from the previous discussion, work with your grade-level teams to:

1) Identify the learning goal(s) for the lesson.

2) Discuss how you anticipate students will respond in the lesson.

3) Plan possible probing questions to help advance student thinking in the lesson.

Purposeful Planning:
***Anticipating and Monitoring* Discourse**

Lesson:		
Step 1: Identify the learning goal(s) for the lesson. Record the learning goal(s) below.		
Learning Goal(s): *Students will….*		
Step 2: Anticipate student responses to the lesson. Remember to: • Consider which student strategies will be most useful in addressing the content. • Consider which strategies students are likely to use. • How to respond to likely student work.	**Step 3**: Plan probing questions to get students back on track or advance student understanding. Use these questions in class during the monitoring phase of the discussion.	**Step 4:** Select work to be shared and sequence responses in a specific order for discussion. What strategies will you highlight to address the learning goals?
Sample strategy:	Purposeful questions to respond:	Student name: In what order will you share this student work? What part of the student work will you highlight?
Sample strategy:	Purposeful questions to respond:	Student name: In what order will you share this student work? What part of the student work will you highlight?

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Reflecting on purposeful planning

How does purposeful planning help you attend to the components of rigor?

What specific instructional strategies did you focus on?

Points to Ponder

What changes did you make to the EngageNY lesson(s)? Why?

How did your conversation help determine a plan to fill gaps in learning?

Math Content Module 5, Session 4
Grades 6-9

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MATHEMATICS CONTENT MODULES

Module 5: Recognizing and generating equivalent expressions

Module 5 Closing slides

Grades 6-9

References

Key resources available via the Louisiana Department of Education web page — K-12 Math Planning Resources, https://www.louisianabelieves.com/resources/library/k-12-math-year-long-planning — include the following:
K–12 Louisiana Student Standards for Mathematics (LSSM)
Teachers Companion Documents
Implement The Eureka Curriculum: Louisiana Eureka Guides.
Help Students Who Struggle: Remediation Guides.
The Louisiana Student Standards for Mathematics: A Guide to Rigor in Mathematics 2.0, at https://www.louisianabelieves.com/docs/default-source/year-long-planning/k-12-lssm-alignment-to-rigor.pdf This document (name in link: "K-12 LSSM Alignment to Rigor") as well as the individual rigor documents for each grade level—e.g., "Kindergarten LSSM Alignment to Rigor," "Grade 1 LSSM Alignment to Rigor," and so on) can be downloaded via this web page: https://www.louisianabelieves.com/resources/library/k-12-math-year-long-planning

Other resources referenced include:
EngageNY, from the New York State Department of Education
Various curriculum resources, available via https://www.engageny.org
Pre-Kindergarten-Grade 5 Mathematics: Curriculum Map and Guiding Documents. EngageNY. Available via https://www.engageny.org/resource/pre-kindergarten-grade-5-mathematics-curriculum-map-and-guiding-documents
These curriculum resources are shared under the EngageNY terms of use (https://www.engageny.org/terms-of-use) and the Creative Commons license—Attribution-NonCommercial-ShareAlike 3.0 Unported (CC BY-NC-SA 3.0)—mentioned therein: https://creativecommons.org/licenses/by-nc-sa/3.0

Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions.* Reston, VA: National Council of Teachers of Mathematics. Available at http://www.nctm.org/Store/Products/5--Practices-for-Orchestrating-Productive-Mathematics-Discussions

Math Content Module 5
Grades 6-9
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What to expect in Module 6

- **Deepening Mathematical Content Knowledge** for effective instruction
- **Exploring Coherence** in the LSSM
- **Key Shifts in Action** through an EngageNY lesson
- **Promoting Rigor** in the classroom
- **Instructional Strategies** for implementing the EngageNY curriculum
- The role of a **Productive Classroom Culture** in the EngageNY curriculum
- **Purposeful Planning** of the EngageNY curriculum

Math Content Module 5
Grades 6-9
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Module evaluation

Your feedback and comments are important to us. Please complete the module evaluation found at:

http://www.utdanacenter.org/ldoe/surveys