Math Module 4 Overview
Module Title: Extending and Applying Operations with Rational Numbers
Grade Band: 6–9

In Module 4, participants develop a common understanding of extending and applying operations with rational numbers.

In Session 1, participants deepen their knowledge of content related to the big idea of extending and applying operations with rational numbers. Participants explore the coherence evident in the LSSM related to this big idea across grade 6–9.

Session 2 is a continuation to the *5 Practices for Orchestrating Productive Mathematics Discussions*, with a focus on the second practice of monitoring student responses. Participants learn that in order for teachers to effectively monitor students' work, it is crucial that teachers plan purposeful questions to support student thinking while students are working on the task. Participants revisit the mathematical task from Session 1 to plan probing questions to anticipated student responses. Participants recognize incorporating purposeful questions in lessons will help move student thinking towards intended learning goals.

In Session 3, participants gain clarity on how the content standards manifest in the secondary classroom and how they look when executed well in curriculum and instruction by experiencing a grade 6 EngageNY lesson. Participants unpack the structure of the EngageNY lesson connected to the mathematics learning experience and identify implications for planning and delivering high-quality, standards-aligned mathematics instruction.

In Session 4, participants revisit the *Planning Guide* in order to connect a foundational study of grade-level or course standards for the Module 4 focus content of extending and applying operations with rational numbers and to plan a related EngageNY lesson for their grade-level or course.

Module 4 Key Understandings

As a result of Module 4, participants will know:

- How the standards move from extending and applying operations with rational numbers across grade 6–9.
- Which math tasks and lessons from EngageNY and other Tier 1 resources are related to the focus content of extending and applying operations with rational numbers.
- How to monitor students' work on, and engagement with, a mathematical task.
- How the incorporation of purposeful questions in lessons will help move student thinking towards intended learning goals.
- A process for high-quality collaborative planning aligned to standards.
- How available resources can be used to support teaching and learning of the LSSM content and practice standards.

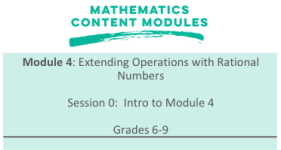
Module 4: Facilitation Recommendations	
Session Length	Six-hour module including: Session 0: Setting the Stage for Module 4 (5 min) Session 1: Deepening Mathematical Content Knowledge and Exploring Coherence in the LSSM (85 min) Session 2: Instructional Strategies to Improve Curriculum Implementation (90 min) Session 3: Key Shifts in Action Through an EngageNY Lesson (120 min) Session 4: Purposeful Planning of the EngageNY Curriculum (60 min) Plus: Module 4 Closing Slides and Participant Evaluation Survey (15 min)
Session Pairing Options	The recommended facilitation of Module 4 is to be done as a full six-hour day of professional learning, with all six sessions facilitated back to back. Facilitators should plan for participants to take the Module 4 participant evaluation survey in Session 4, preferably before leaving the session.
Time of Year	If possible, within four weeks of experiencing Module 3

Facilitation Recommendations	
Session Length	Six-hour module and additional 20 minutes for opening and closing slides
Time of Year	If possible, within four weeks of experiencing Module 3

Preparing to Facilitate *What materials and supplies are needed to facilitate this session? What must the facilitator do to prep for the actual session?*	

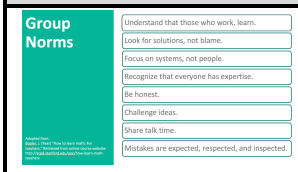
Session Materials:
• Print facilitator and participant handouts (see handout **M4_CL-01_Module 4 Posters and Handouts Guide**).
 	
Additional Supplies:
• Chart paper
• Markers
• Sticky notes
	Facilitator Pre-Work and Prep
	• Create and hang the following posters:
	  ○ Parking Lot
	  ○ Group Norms

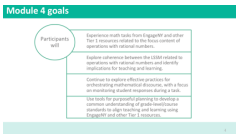
Time & Slide Thumbnail	Facilitator Notes Session 0	Materials


Slide 1
Time Allotted: 2 minutes
	Session 0: Setting the Stage for Module 4
	Slides 1–4: 5 minutes
	
	Critical Idea
	• It is important to set a positive tone for learning.
	
	Step-By-Step Instructions
	• Before the start of the session, use handout **M4_CL-01_Module 4 Posters and Handouts Guide** to organize your preparation. It lists the posters and other materials you will need to prepare before the session as well as all handouts needed for this module.
	• As participants come in for the day, ask them to sit in mixed grade-level groups of four.
	• Welcome the group to Module 4 and if necessary, introduce yourself.
	• Direct participants to handout **M4_P-00 Module 4 Presentation Slides**. Say, "Please use handout M4_P-00 throughout each of the sessions in Module 4. You may use the handout to follow along with the presentation slides and as a place to record your notes and responses to specific activities and reflection questions."
	• Remind participants of the Parking Lot poster to participants. This is a place where they can leave questions throughout the day. Explain to them that periodically throughout the day, you will be answering any questions on the Parking Lot poster.
	• Take care of "housekeeping" items before beginning the day:
	  ○ Scheduled start and stop times
	  ○ Breaks
	  ○ Lunch
	  ○ Location of restrooms
	  ○ Silencing of cell phones
	  ○ Orienting participants to the handouts for the day, including the handout of the slides.
	
	Words of Wisdom
	• N/A
		M4_CL-01_Module 4 Posters and Handouts Guide
		
		M4_P-00 Module 4 Presentation Slides

 Slide 2 Time Allotted: < 1 minute	***Critical Idea*** • For participants to buy in to the project, it is important to provide them with the goal and purpose. ***Step-By-Step Instructions*** • Say, "Recall this slide from previous modules. The work in which you are engaged is part of a bigger picture. The overarching goal of the initiative is to improve the quality of instruction in math classrooms by developing deep mathematical content knowledge of teachers and connecting that content knowledge to the practical implementation of a quality curriculum." ***Words of Wisdom*** • This slide provides participants with the goal of the three-year initiative.	

Time & Slide Thumbnail	Facilitator Notes Session 0	Materials


Slide 3
Time Allotted: 2 minutes
	Critical Point
	• Participants must agree to honor these norms, which will help provide an environment for professional conversation.
	
	Step-By-Step Instructions
	• Say,
	  ○ "In order to meet the goals and learning expectations of the initiative, we have group norms that will continue to guide our collaborative work."
	  ○ "Please independently read the slide and choose one of the norms to which you will pay particular attention throughout Module 4."
	• As a way to encourage accountability, direct participants to tell their tablemates which norm they selected.
	
	Words of Wisdom
	• This slide sets the tone for how we will go about our work for the day.
	
	Image credit: Chad Baker/Jason Reed/Ryan McVay/Photodisc/ Thinkstock (item number AA014176)	

 Slide: 4 Time Allotted: 1 minute	***Critical Point*** • The lens for learning is beginning with the end in mind. ***Step-By-Step Instructions*** • Say, "Please quietly read the learning expectations for Module 4." • After participants have finished reading, explain the following. • Say, "In Module 4, you will consider the standards related to extending operations with rational numbers and engage in tasks and a lesson from EngageNY related to this content. You will also explore how curricular resources can be used for purposeful planning to develop conceptual understanding, procedural skill and fluency, and application skills with your students." ***Words of Wisdom*** • This slide should be brief and is about clarifying the learning expectations of Module 4.	

Math Module 4, Session 1 Overview

Session Title: Deepening Mathematical Content Knowledge and Exploring Coherence in the LSSM

Grade Band: 6–9

Session Overview *What are the overarching goals of this session? Why are those goals important? How do they connect to what's come before and what will come after?*	
Participants engage in a deep study of grade-level standards related to operations with rational numbers by working on a rich task and identifying related standards. The discussion of the standards builds on the ideas introduced in the Vertical Alignment discussion from a previous module but allows teachers to work in grade-level teams to dig deeper into their grade level's role in the alignment to ensure that teaching and learning are focused on the intended learning as described in the standards.	
Session Key Points *What are the 2-4* most *important takeaways from this session?*	**Session Products** *What will participants produce during the session? Immediately following the session?*
Participants will know… • Students' work with operations with rational numbers begins in 4th grade and all operations with rational numbers are complete by 7th grade. In 8th grade and Algebra I, the focus is on applying operations with rational numbers to solve equations. • In high school, students extend their knowledge of operations with rational numbers to operations with rational expressions.	• Poster summarizing operations with rational numbers at each grade level • Summary of the vertical progression of operations with rational functions from grade 4 to grade 11

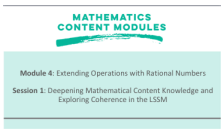
Session Objectives *What will participants know, believe, and be able to do as a result of this session?*	**Session Agenda at a Glance** *What is the overall flow of this session?*
Participants will be able to… ● Describe the vertical progression of operations with rational numbers across grades. (CK) ● Articulate how their new understanding of the standards impacts instruction at their grade level. (CK, CP) CK: Content Knowledge CP: Content Pedagogy CS: Curriculum Skills	• Engage in Egyptian Fractions II task. • Study the standards. • Discuss vertical progression. • Reflect on the session.

Facilitation Recommendations	
Session Length	85 minutes
Time of Year	If possible, within four weeks of experiencing Module 3.
Session Pairing Options	It is strongly recommended that this session be delivered in full in one session. If it is not possible to deliver in one session, it is important to have the split sessions in consecutive days. If necessary, you can split the session after Slide 7.

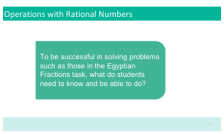
Preparing to Facilitate *What materials and supplies are needed to facilitate this session? What must the facilitator do to prep for the actual session?*	
Session Materials: ● Handout **M4_P-01 Egyptian Fractions II** ● Handout **M4_P-02 Operations with Rational Numbers Across the Grade Levels** **Additional Supplies:** ● Chart paper ● Chart markers ● Sticky notes	**Facilitator Pre-Work and Prep:** • Study the Egyptian Fractions II task using **M4_CL-02 Egyptian Fractions II Commentary**. • Create Upper Elementary School and Upper High School summary posters (see **M4_CL-01 Module 4 Posters and Handouts Guide**).

Participant Pre-Work *What should participants bring to this session? What should they do or prepare in advance of this session?*	
Participants must bring: ● LSSM standards	**Before this session, participants should:** • N/A

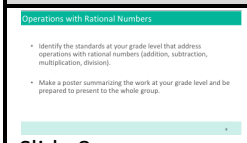
Time & Slide Thumbnail	Facilitator Notes Session 1	Materials

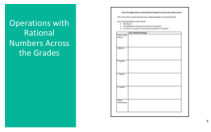

 	
Slide 5
Time Allotted: 1 minute
	Session 1: Deepening Mathematical Content Knowledge and Exploring Coherence in the LSSM
	Slides 5–10: 85 minutes
	
	Critical Idea
	• Introduce the session.
	
	Step-By-Step Instructions
	• Say,
	  ○ "As you have noticed, each module focuses on a big idea in mathematics. In previous modules, you were introduced to processes to study the standards, see the coherence in the standards, and guide your planning so that instruction is aligned to the standards. In this module, we will study standards related to the big idea of extending operations with rational numbers and make use of the study and planning processes we experienced in earlier modules.
	  ○ "Session 1 is designed for the purpose of deepening your own mathematical knowledge for effective instruction. The EngageNY curriculum has similar math learning activities built into its lessons, although they may be spread across several lessons and not necessarily be presented in the same way we are about to experience. It is important to keep in mind that you may experience some a-ha moments in this section that will not come as quickly and as easily to your students."
	
	Words of Wisdom
	• Reiterate throughout Module 4 that it is important for participants to abide by the pacing of the EngageNY curriculum, which is designed to be more developmentally appropriate for the students. The purpose of the opening math task is to get participants thinking about content related to the big idea of this module.	

  Slide 6 Time Allotted: 30 minutes	***Critical Idea*** • Teachers will engage in math tasks aligned to grade-level standards related to the big idea of extending operations with rational numbers. ***Step-By-Step Instructions*** • Verify that teachers are sitting in mixed grade-level groups of no more than 4. • Say, "To get you thinking about the big idea of this module, let's do some math! You will do a task related to the big idea. This task was modified from an Illustrative Math task. This task is called Egyptian Fractions II and involves working with rational numbers. The ancient Egyptians represented all fractions as the sum of unit fractions; this sum of unique unit fractions is what we call an 'Egyptian Fraction.' In this task, you will work with your grade-level group to find Egyptian Fractions." • Distribute handout **M4_P-01 Egyptian Fractions II** to participants and have them read the first paragraph silently to understand what an Egyptian Fraction is. When participants have finished reading, popcorn out comments from the whole group to help clarify the meaning of an Egyptian Fraction. Key points to emerge are: ○ All fractions are written as a sum of unit fractions.	**M4_P-01_Egyptian Fractions II** **M4_CL-02 Egyptian Fractions II Commentary**

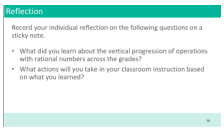
Time & Slide Thumbnail	Facilitator Notes Session 1	Materials
	○ The unit fractions in the sum must be distinct. ○ To make sure all participants understand the task, ask, "Is $\frac{3}{7}=\frac{2}{7}+\frac{1}{7}$ an Egyptian Fraction? Why or why not?" Point out that this sum uses $\frac{2}{7}$, which is not a unit fraction. ○ Ask, "Is $\frac{3}{7}=\frac{1}{7}+\frac{1}{7}+\frac{1}{7}$ an Egyptian Fraction? Why or why not?" Point out that the unit fractions used in the sum are not distinct. • Say, "You will have 15 minutes to work on the task. For the first 5 minutes, work independently to make sense of the problem and begin forming a strategy. After 5 minutes, I will direct you to work with your group. You can share your thoughts and continue working on the problem with your group." • Watch for groups who struggle with proving the identity in question 2a algebraically. Instead, encourage these groups to discuss question 2b. They will use the identity to verify their work in questions 1a and 1b. • Circulate around the room while participants are working, finding a few different methods of solving the problem. As groups finish working, ask participants from two or three groups to share with the whole group their solutions and methods. ***Words of Wisdom*** • Be ready for teachers to struggle with this task. Make connections back to the discussion in Module 1 related to classroom culture and growth mindsets to encourage participants to persist through their struggles, just as they would expect their students to do in the classroom. • Use the handout **M4_CL-02 Egyptian Fractions II Commentary** to anticipate ways to support participants who are struggling with the task. Image credit: pius99/iStock/ Thinkstock (item number 504820544)	
 Slide 7 Time Allotted: 7 minutes	***Critical Idea*** • At this point, participants will start thinking about students' experiences with operations with rational numbers across the grades. ***Step-By-Step Instructions*** • To get the conversation started, ask participants to think about the Egyptian Fractions II task they just completed and the question on this slide. • Give participants 2 minutes to respond to the question in their groups. Then chart some responses on chart paper. ***Words of Wisdom*** • Be sure to highlight responses to the question on the slide that speak to the knowledge and skills that students must develop to be successful with the second question of the Egyptian Fractions tasks.	Chart paper
	Critical Idea • It is important for teachers to understand how the math at their grade level fits into the development of a mathematical idea across grade levels. Teachers move	LSSM standards

Time & Slide Thumbnail	Facilitator Notes Session 1	Materials


Slide 8
Time Allotted: 20 minutes
	from studying a task to looking at standards at their own grade level to see the coherence in the standards across grades.
	
	Step-By-Step Instructions
	• Participants will now study the standards related to operations with rational numbers in same grade-level groups. Have teachers move their belongings and form same grade-level groups of no more than 4. Depending on the makeup of the participants, there may be more than one grade-level group.
	• After teachers form their new groups, say, "Now that you have thought deeply about the mathematics related to operations with rational numbers, let's look at how the standards prescribe how and when students are operating with rational numbers across the grades."
	• Say, "In your groups, study your grade-level standards and identify those related to operations with rational numbers. Look for work with all four operations (addition, subtraction, multiplication, and division). Also, remember that rational numbers are both positive and negative and include fractions, decimals, and integers. Make a poster summarizing the work at your grade level. Your poster should include both the standard coding and summary statements that describe what students should know and be able to do with operations with rational numbers by the end of the year at your grade level. Be ready to present your findings to the whole group."
	• Give participants 15 minutes to study their standards and create their posters.
	• As groups start working, hang the pre-prepared posters for upper elementary and upper high school on the wall, leaving space in the middle for 4 posters for grades 6–9.
	• Monitor groups as they work. Some things to watch for in each grade level are:
	  ○ Grade 6 – Although participants should focus their attention on the standards dealing with operations (6.NS.A.1, 6.NS.B.2-3), they might notice that in grade 6 students are introduced to negative numbers (see 6.NS.C.5-8). Students do not operate with negative numbers in grade 6, but their understanding of negative numbers is important to the 7th grade standards in which students operate with positive and negative rational numbers.
	  ○ Grade 7 – The entire Number System domain in grade 7 deals with operations of rational numbers. You may want to point teachers to 6.NS.C.5 so that 7th grade teachers see that students have been introduced to the concept of a negative number in grade 6, but have not learned anything about operations with negative numbers. You might also want to point out that no standards in grade 8 deal with the skill of operating with rational numbers. It is critical for the 7th grade teacher to know that all operations with rational numbers are finalized in grade 7, including positive fractions, negative fractions, and integers.
	  ○ Grade 8 – Notice there are no standards in grade 8 that deal with the skill of operating with rational numbers. Instead, students must apply their knowledge of operations with rational numbers to solve linear equations (see 8.EE.C.7b). It is also interesting to note that in grade 8, students learn about irrational numbers (see 8.NS.A.1-2). While they are not operating with irrational numbers, they are introduced to this new type of number. One other application of operations with rational numbers is found in 8.EE.A.4 where students operate with numbers expressed in scientific notation. It is
		Pre-made Upper Elementary and Upper High School summary posters

Time & Slide Thumbnail	Facilitator Notes Session 1	Materials
	interesting to note that this is the only place scientific notation appears in the standards. ○ Algebra I – Notice there are no standards in grade 8 or Algebra I that deal with the skill of operating with rational numbers. Instead, in Algebra I, like in grade 8, students apply their knowledge of rational numbers. In A1: N-RN.B.3, students analyze the sum and product of rational and irrational numbers. In A1: A-REI.B.4, students apply knowledge of rational numbers to complete the square and derive the quadratic formula and solve quadratic equations. In A1: A-APR.A.1, students understand that polynomials form a system analogous to integers and explore the closure property as it relates to the operations of addition, subtractions and multiplication of polynomials. ○ They also continue the work from 8th grade in solving more complex linear equations, linear inequalities, and systems of equations. ***Words of Wisdom*** • Make sure that each poster summarizes the work done at each grade level. Teachers should not copy the standards verbatim. The clarity and quality of these posters will impact the next activity.	
 Slide 9 Time Allotted: 20 min	***Critical Idea*** • It is important for teachers to understand how the math at their grade level fits into the development of a mathematical idea across grade levels. ***Step-By-Step Instructions*** • Have groups post their posters in order from grades 6–9 in between the upper elementary and upper high schools posters. If there are multiple groups per grade level, have them stack their posters vertically. Remind groups that they need a representative to present their groups' findings. • Distribute the handout **M4_P-02 Operations with Rational Numbers Across the Grade Levels**. • Say, "Before the session, I created posters for upper elementary and upper high school like the ones you created for your grade levels. We will now hear presentations for each grade level so that we can come to a common understanding of the vertical progression of operations with rational numbers from grade 4 to grade 11. Use the handout M4_P-02 to capture key understandings about the standards at each grade level as you listen to the group presentations." • After all posters are hung, summarize the upper elementary poster for the group. Then have groups present from grades 6 to 9. End with a summary of the upper high school poster. Limit presentations to 2-3 minutes per grade level. ***Words of Wisdom*** • If you have multiple groups at each grade level, you need to carefully manage the time for presentations. Encourage duplicate grade level groups to just share out ideas that have not yet been shared for their grade level. If you can save time in this section, you will have more time for discussion in the reflection activity in the next section.	**M4_P-02 Operations with Rational Numbers Across the Grade Levels**

Time & Slide Thumbnail	Facilitator Notes Session 1	Materials


Slide 10
Time Allotted: 7 minutes
	Critical Idea
	• Teachers need to reflect on their learning and consider how their new learning impacts instruction in their classroom.
	
	Step-By-Step Instructions
	• Give participants 2 minutes to reflect individually using the questions on the slide and record their reflections on sticky notes.
	• Have participants pair up to share their reflections. You can use an eye-contact, same-colored shirt, or some other protocol to quickly pair up participants. Give pairs 2 minutes to share their individual reflections. Then repeat this process as time allows.
	• As time allows, ask participants to share with the whole group interesting comments they heard. Summarize the session, making sure these key points emerge:
	  ○ Students' work with operations with rational numbers begins in 4th grade as they learn to add and subtract fractions with like denominators.
	  ○ By 7th grade, all operations with all rational numbers, both positive and negative, is complete.
	  ○ In 8th grade and Algebra I, the focus is on applying operations with rational numbers to solve equations.
	  ○ In high school, students extend their knowledge of operations with rational numbers to operations with rational expressions.
	  ○ Students' success with rational functions and expression in upper high school depends on their conceptual understanding of operations with rational numbers built in earlier grades.
	  ○ Understanding the coherence in the standards can help teachers plan appropriately to address gaps in student learning. The Louisiana Remediation Guide and the Student Achievement Partners coherence maps can help teachers identify related standards to refer back to.
	• Post two sheets of chart paper, titled "New Learning" and "Action Items." As participants finish this activity, ask them to post their sticky note reflections on the appropriate poster.
	
	Words of Wisdom
	• Teachers should pair up quickly so that they can have the benefit of talking with several partners during this reflection activity.
		Sticky notes
		
		Two pieces of blank chart paper titled: 1) "New Learning" and 2) "Action Items"

Math Module 4, Session 2 Overview

Session Title: Instructional Strategies to Improve Curriculum Implementation

Grade Band: 6–9

Session Overview *What are the overarching goals of this session? Why are those goals important? How do they connect to what's come before and what will come after?*
Session 2 is a continuation of the *5 Practices for Orchestrating Productive Mathematics Discussions*, with a focus on the second practice of monitoring student responses. The remaining practices will be explored in depth in Modules 5 and 6. Participants learn that in order for teachers to effectively monitor students' work, it is crucial that teachers plan purposeful questions to support student thinking while students are working on the task. Participants analyze the EngageNY lesson from Session 2 to plan probing questions to anticipated student responses. Participants recognize that incorporating purposeful questions in lessons will help move student thinking towards intended learning goals.

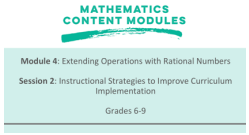
Session Key Points *What are the 2-4* most *important takeaways from this session?*	**Session Products** *What will participants produce during the session? Immediately following the session?*
Participants will know: • Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships. • While ***monitoring*** students' work, probing questions should be used to keep students focused on and moving forward in their understanding of the task and the mathematics involved.	• Completed handout **M4_P-09 Anticipating and Monitoring Discourse** with anticipated student responses within an EngageNY lesson and a list of probing questions in response to these anticipated responses

Session Objectives *What will participants know, believe, and be able to do as a result of this session?*	**Session Agenda at a Glance** *What is the overall flow of this session?*
Participants will be able to… • Plan and use purposeful questions to assess and advance students' reasoning and sense making towards intended learning outcomes. (CP, CS) CK: Content Knowledge CP: Content Pedagogy CS: Curriculum Skills	• Review the *5 Practices for Orchestrating Productive Mathematics Discussions.* • Explore the second practice, ***monitoring*** student responses. • Do the questions card sort. • Define four types of questions. • Anticipate student responses and plan probing questions for sample EngageNY lesson. • Reflect on the session.

Facilitation Recommendations	
Session Length	90 minutes

Preparing to Facilitate *What materials and supplies are needed to facilitate this session? What must the facilitator do to prep for the actual session?*	
Session Materials: • Handout **M4_P-01 Egyptian Fractions II** • Handout **M4_P-03 Posing Purposeful Questions – Question Types** • Handout **M4_P-04 Anticipating and Monitoring Discourse** • Handout **M4_CL-03 Sample Questions Card Set** **Additional Supplies:** • Sticky notes	**Facilitator Pre-Work and Prep:** • Print participant handouts (1 per participant). • Complete handout **M4_P-04 Anticipating and Monitoring Discourse** to prepare for facilitation.

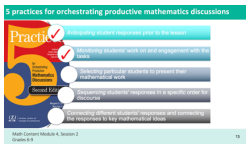
Participant Pre-Work *What should participants bring to this session? What should they do or prepare in advance of this session?*	
Participants must bring: • Handout **M4_P-01 Egyptian Fractions II**	**Before this session, participants should:** • Complete handout **M4_P-01 Egyptian Fractions II**

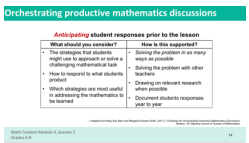
Time & Slide Thumbnail	Facilitator Notes Session 2	Materials
 Slide 11 Time Allotted: < 1 minute	**Session 2: Instructional Strategies to Improve Curriculum Implementation** **Slides 11–19: 90 minutes** ***Critical Idea*** • This slide is an introductory slide for Session 2. ***Step-by-Step Instructions*** • Tell participants that we will transition into Session 2. ***Words of Wisdom*** • Participants should be sitting in mixed-grade level groups for Session 2. This session will be more effective if participants are able to sit with other participants they have not worked with yet.	

Slide 12
Time Allotted: 5 minutes
	Critical Idea
	• Emphasize that the purpose of this section is to give participants an overview of the model for facilitating mathematics discourse described in the resource *5 Practices for Orchestrating Productive Mathematics Discussions*, and to take a deeper look at the second practice: **monitoring**. The other practices described in this book will be explored in depth in other modules.
	
	Step-By-Step Instructions
	• Say,
	  ○ "Recall the session about discourse from Module 4. To promote productive student discourse in the mathematics classroom, we will continue to examine the '5 practices model' as originated by Margaret Smith and Mary Kay Stein. Smith and Stein's research laid out the framework as a process for using student thinking and student work as the launch pad for rich discussions that get at the heart of important learning."
	  ○ "The first iteration of this model was done with mathematics; later, the model was expanded to also address science discourse. We will use this resource as the foundation of our learning today."
	  ○ "The purpose of this section of learning today will be to review the 5 practices and to explore the second practice in the model, **monitoring.**
	
	Words of Wisdom
	• The 5 practices model is very rich. Participants can become overwhelmed, so strive to keep the overview of the 5 practices brief.
	
	Citation:
	Smith, M. S., & Stein, M. K. (2011). 5 practices for orchestrating productive mathematics discussions. Reston, VA: National Council of Teachers of Mathematics. Available at http://www.nctm.org/Store/Products/5--Practices-for-Orchestrating-Productive-Mathematics-Discussions	

	Critical Idea • Emphasize that the purpose of this section is to give participants a quick review of the model for facilitating mathematics discourse described in the resource *5 Practices for Orchestrating Productive Mathematics Discussions,* and to take a	

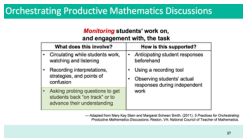
Time & Slide Thumbnail	Facilitator Notes Session 2	Materials

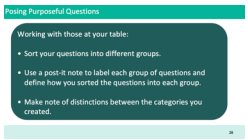

Slide 13
Time Allotted: 3 minutes
	deeper look at the first practice: **monitoring**. The other four practices described in this book will be explored in depth in other modules.
	
	Step-By-Step Instructions
	• Say,
	  ○ "Remember, this visual is an overview of the *5 Practices for Orchestrating Productive Mathematics Discussions*."
	  ○ "The 5 practices model is a process of planning and instruction that teachers can implement for promoting productive discussions in classrooms."
	  ○ "The EngageNY curriculum resource makes use of these 5 practices in many lessons."
	  ○ "By taking the time to understand the model fully, you should be able to better implement the EngageNY curriculum to its fullest intent."
	• **<u>Animate the slide for each step.</u>**
	• Briefly review the description for each step:
	  ○ "This process depends on the foundation of establishing a clear learning goal for discussions, which often EngageNY does for you."
	  ○ "The first practice, ***anticipating* student responses** to a challenging task, is part of the planning that occurs BEFORE students engage in the mathematics task."
	  ○ "The second practice, ***monitoring* student work** during engagement with the task, happens while students are working on the task, either individually, in pairs, or in small groups."
	  ○ "The third practice, ***selecting* particular student work** samples (or students, or groups of students, to present their mathematical work), happens after students have completed the task."
	  ○ "The fourth practice, ***sequencing* student work** in a specific order for discussion, also happens after students have completed the task."
	  ○ "The fifth and final practice, ***connecting* student work samples** to each other and to the key learning goal (connecting different students' responses, and connecting the responses to key mathematical ideas), happens after a sequence is determined."
	• **<u>Animate the slide.</u>**
	• Say, "In the previous module, we explored the practice of anticipating student responses."
	• **<u>Animate the slide</u>.**
	• Say, "The focus of our session today will be on ***monitoring* student work**."
	
	Words of Wisdom
	• While EngageNY does make use of many of these practices, it does not identify any of them specifically. Therefore, it is important that teachers using the EngageNY curriculum understand how to recognize each of these practices in order to best meet the intent of the lesson.	

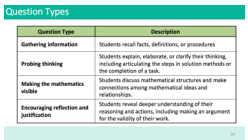
 Slide 14	***Critical Idea*** • This is a quick review of the first practice, anticipating student responses. The first practice, ***anticipating***, was shared in Module 3. Participants will apply this knowledge later in the session to ***anticipate*** and ***monitor*** student responses to the Egyptian Fractions task from Session 1.	

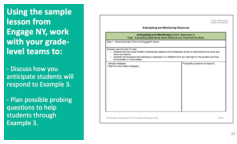
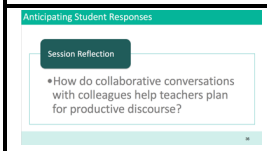
Time & Slide Thumbnail	Facilitator Notes Session 2	Materials

Time Allotted: 2 minutes	***Step-By-Step Instructions***
	• Do a brief refresh of the first practice, ***anticipating***. Say,
	  ○ "This slide is a summary of what you learned in Module 3. You will apply this learning later in the session to anticipate and monitor student responses."
	  ○ "Recall the first practice of ***anticipating*** student responses. Remember, there are three important aspects of ***anticipating***."
	  ○ "In mathematics, when teachers consider the most useful strategies that students are likely to use, they are able to consider the challenges that students might face and what can be done about them."
	  ○ "Teachers will need to consider how students might approach an instructional task. This consideration requires that teachers engage in the task or activity themselves and consider different ways it could be approached or interpreted. They should consider the various representations that students might use in answering questions and the common errors that students make. They should also consider:
	  ▪ how students might interpret a problem,
	  ▪ the strategies they may use, both correct or incorrect, and
	  ▪ how their strategies might relate to the learning goal/s that have been established."
	  ○ "Teachers will need to consider how they will respond to the strategies that students choose and the answers they generate. By taking into consideration how students will work through a problem, teachers can begin to generate questions that can be used to help students deepen their thinking. This reduces the amount of time that teachers will need to spend reacting in the moment."
	  ○ "After ***anticipating*** is the second practice, ***monitoring*** student work. We will learn more about that now."
	
	Words of Wisdom
	• This slide is a quick refresher of the first practice of anticipating student responses. Keep this brief so participants have enough time to analyze the second practice of monitoring.	


Slide 15
Time Allotted: 7 minutes
	Critical Idea
	• Actively monitoring students as they work on a task yields an abundance of data about students' current understanding of the mathematics within the task. This data is valuable for facilitating productive mathematical discourse.
	
	Step-By-Step Instructions
	• Say, "Take a quick minute to review the slide and share your understanding of the process of ***monitoring*** with those at their table."
	• After giving participants a couple of minutes to talk with those in their groups, share the following with them:
	  ○ "This second process, ***monitoring***, happens while students are working on the task, either individually, in pairs, or in small groups."
	• **<u>Animate the colored box.</u>**
	  ○ "While ***monitoring*** students' work, the teacher should ask probing questions to keep students focused on and moving forward in their understanding of the task and the mathematics involved."
	  ○ "Therefore, it is important to plan some of those questions ahead of time,	

Time & Slide Thumbnail	Facilitator Notes Session 2	Materials
	keeping in mind the set of strategies that your team anticipates will be used by students." ***Words of Wisdom*** • Clarify for participants that monitoring students' work on a task happens in real-time in the classroom. However, monitoring is more effective if participants purposefully plan probing questions before the lesson, and in response to anticipated student strategies.	
 Slide 16 Time Allotted: 15 minutes	***Critical Idea*** • Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships. ***Step-By-Step Instructions*** • Hand out a set of question cards, handout **M4_CL-03 Sample Questions Card Set**, to each table group of 4 participants. • Participants should work together to classify the questions into different categories. They should use a separate sticky note to label each category. • Have groups continue to sort additional questions into groups and name any additional categories that emerge. • Once small groups are done categorizing, debrief as a whole group by choosing a volunteer to explain the categories they created. Encourage the participants to justify the categories and clarify the differences between each of the categories identified. • Ask the remaining small groups to identify a similarity or difference in the categories they created by asking 1 or 2 more volunteers to briefly elaborate or expand on the initial groups' thinking. ***Words of Wisdom*** • While groups are working, monitor group conversations and make note of the different categories identified. Also, make note of connections to questioning patterns (funneling and focusing questions) and use as a transition into the discussion on the next slide. Modeling this process will demonstrate facilitation of the anticipating and monitoring stages of the 5 practices.	**M4_CL-03 Sample Questions Card Set**


 	
Question Type | Description
Gathering information | Students recall facts, definitions, or procedures
Probing thinking | Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task.
Making the mathematics visible | Students discuss mathematical structures and make connections among mathematical ideas and relationships.
Encouraging reflection and justification | Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.
Slide 17
Time Allotted: 10 minutes
	Critical Idea
	• Different question types will elicit responses that are crucial to mathematics teaching and learning.
	
	Step-By-Step Instructions
	• Direct participants to handout **M4_P-03 Posing Purposeful Questions – Question Types.** Take a few minutes to examine the four question types, their descriptions, and examples.
	• Share the following with participants:
	  ○ There are a number of frameworks that categorize the types of questions that teachers ask.
	  ○ Although the categories differ across frameworks, there are commonalities among the types of questions.
	  ○ *Principles to Actions* gives us research-based insights into four different types
		M4_P-03 Posing Purposeful Questions – Question Types

Time & Slide Thumbnail	Facilitator Notes Session 2	Materials
	of questions that each meet specific purposes in the mathematics classroom. ○ The chart on the slide displays four question types matched to the key aspects that are important for mathematics teaching and learning. ○ Although the question types differ with respect to the level of thinking required in a response, all of the question types are necessary in the interactions among teachers and students. • Elicit any additional comments and discuss in whole group. ***Words of Wisdom*** • All of the question types are necessary in the interactions among teachers and students. Incorporating purposeful questions in lessons will help move student thinking towards the learning goal.	
 Slide 18 Time Allotted: 45 minutes	***Critical Point*** • Use the Egyptian Fractions task from Session 1 to give participants a context for ***anticipating*** and ***monitoring*** student responses. ***Step-By-Step Instructions*** • For this task, participants will be working in pairs to practice the ***anticipating*** and ***monitoring*** stages of the *5 Practices for Orchestrating Productive Mathematics Discussions*. • Give the following directions. Say, ○ "Locate handout **M4_P-04 Anticipating and Monitoring Discourse."** ○ "In pairs, revisit the Egyptian Fractions task from Session 1. Examine specifically Example 6. Record 3 or 4 possible strategies you anticipate students will use to solve this problem." ○ "Next, identify which solutions are most useful in addressing the content and goals of the task." ○ "Then brainstorm at least 3-5 purposeful questions you might use in response to the sample strategies that can support and advance student thinking towards the learning goal. Record your responses on handout **M4_P-04**." ○ "When you are finished, choose 1 sample student strategy and 3-5 purposeful questions and record on chart paper." • After pairs have created their posters, ask them to hang the posters on the wall. • To debrief the activity, have participants do a gallery walk around the room. As they examine the other posters, they should make note of new strategies on handout **M4_P-04.** ***Words of Wisdom*** • Encourage pairs to have a manageable goal of no more than 3 to 4 strategies documented. Looking for more strategies than that can be difficult to manage while monitoring student work.	**M4_P-04 Anticipating and Monitoring Discourse** Chart paper Markers
 Slide 19 Time Allotted: 3	***Critical Idea*** • Adults do not learn from experience but rather from reflecting on experience. ***Step-By-Step Instructions*** • Say, "The previous activity was less about how to teach rewriting expressions in different forms and more about ways of anticipating how students will engage with the task to prepare for how to move students forward on reaching the learning goal."	

Time & Slide Thumbnail	Facilitator Notes Session 2	Materials

minutes	• Pose the question on the screen. Say, "How do collaborative conversations with colleagues help teachers plan for productive discourse?"
	  ○ Sample responses include:
	  ▪ *When teachers plan for student discourse by establishing learning goals and anticipating student responses to tasks, teachers and students benefit.*
	  ▪ *Teachers are better able to respond to students in ways that support student progress toward the instructional goal, and students will be able to engage in a task that will promote deeper thinking and therefore provide them with the opportunity to explain their thinking to the teacher and their peers.*
	
	Words of Wisdom
	• Collaborative planning and conversations are integral to planning for productive discourse.	

Math Module 4, Session 3 Overview

Session Title: Key Shifts in Action Through an EngageNY Lesson

Grade Band: 6–9

Session Overview *What are the overarching goals of this session? Why are those goals important? How do they connect to what's come before and what will come after?*
The beginning of Session 3 explores the following essential question, "How can EngageNY lessons attend to the components of rigor through engagement of the standards?" Participants experience a grade 6 EngageNY lesson from a student perspective. Intentional teacher moves will support mathematical discourse that builds conceptual understanding and application related to the focus content of extending operations with rational numbers. The EngageNY lesson focuses on standards within the Number System and Number and Quantity domains to deepen participants content knowledge. Immediately following the lesson, participants will unpack the EngageNY lesson related to the learning experience. By analyzing the components of the EngageNY lesson and considering intentional instructional strategies that were used in the facilitation, teachers identify implications for their own planning. A major goal of this session is for teachers to understand how to use the EngageNY lessons to guide their instruction to foster conceptual understanding, procedural skill and fluency, and application skills with students. Teachers consider how mathematical discourse and perseverance in solving problems is used to illuminate the mathematics content. This session sets the stage for Session 4. As teachers identify implications for teaching and learning, it will bring to the surface the need for close analysis of the high-quality curriculum during purposeful planning.

Session Key Points *What are the 2-4* most *important takeaways from this session?*	**Session Products** *What will participants produce during the session? Immediately following the session?*
Participants will know… • The EngageNY teacher materials provide information about standards, including focus standards, foundational standards, and Standards for Mathematical Practice. • How the key shifts in the standards of focus, coherence, and rigor are evident in an EngageNY lesson. • Intentional teacher moves support focus, coherence, and rigor around the mathematics content.	• Record evidence of focus, coherence, and rigor in an EngageNY lesson using handout **M4_P-06 Looking for Evidence of the Key Shifts**. • Engage in the classwork and exercises sections of an EngageNY lesson on handout **M4_P-07_ENY G6 M2 TA L4 Student**.

Session Objectives *What will participants know, believe, and be able to do as a result of this session?*	**Session Agenda at a Glance** *What is the overall flow of this session?*
Participants will… • Gain clarity on how the mathematics standards manifest in the secondary classroom when executed well in curriculum instruction. (CK, CP) • Provide evidence of focus, coherence, and rigor in a model lesson. (CP, CK) • See effective instructional strategies in action that can be implemented directly in their classrooms. (CP)	• Experience EngageNY lesson. • Analyze EngageNY lesson components. • Identify focus content and practice standards identified in the EngageNY lesson plan. • Connect EngageNY lesson plan to the mathematics learning experience. • Identify evidence of the key shifts. • Reflect on the session.

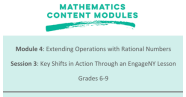
• Identify implications for delivering high-quality, standards-aligned mathematics instruction using the EngageNY resources. (CK, CP) CK: Content Knowledge CP: Content Pedagogy CS: Curriculum Skills	

Facilitation Recommendations	
Session Length	120 minutes
Time of Year	
Session Pairing Options	

Preparing to Facilitate *What materials and supplies are needed to facilitate this session? What must the facilitator do to prep for the actual session?*	

Session Materials:
• Handout **M4_P-00 Module 4 Presentation Slides**
• Handout **M4_P-05 ENY G6 Module 2 Overview**
• Handout **M4_P-06 Looking for Evidence of the Key Shifts**
• Handout **M4_P-07 ENY G6 M2, TA, L4 Student**
• Handout **M4_P-08 ENY G6 M2, TA, L4 Teacher**
• Handout **M4_P-09 Diamond Reflection: Unpacking the ENY Lesson**
 	
Additional Supplies:
• Chart paper
• Markers
• Sticky notes
	Facilitator Pre-Work and Prep:
	• Create an anchor chart with the following terms and definitions:
	  ○ Dividend
	  ○ Divisor
	  ○ Numerator
	  ○ Denominator
	  ○ Quotient
	• Print participant handouts (1 per participant).
	• Study EngageNY lesson handout **M4_P-08 ENY G6 M2, TA, L4 Teacher**.

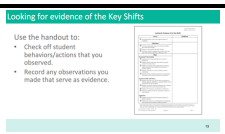
Participant Pre-Work *What should participants bring to this session? What should they do or prepare in advance of this session?*	
Participants must bring: • N/A	**Before this session, participants should:** • N/A

Time & Slide Thumbnail	Facilitator Notes Session 3	Materials
 Slide 20 Time Allotted: 1 minute	**Session 3: Key Shifts in Action Through an EngageNY Lesson** **Slides: 20–33: 120 minutes** ***Critical Point*** • This slide is an introductory slide for Session 3. • The EngageNY lesson participants experience in this session ties directly with the standards they studied in the vertical alignment conversation during Session 1. ***Step-By-Step Instructions*** • Say, "We will now transition into Session 3. As mentioned at the beginning of Module 4, this session is designed for the purpose of deepening your own mathematical knowledge for effective instruction. It is important to keep in mind that you may experience some a-ha moments in this section that will not come as quickly and as easily to your students so it is important to abide by the pacing of the EngageNY curriculum which is designed lead students to the same a-ha moment over the course of multiple lessons, not a single lesson." ***Words of Wisdom*** • Direct participants to sit in mixed grade-level groups of no more than 4. • Reiterate throughout the session that it is important for participants to abide by the pacing of the EngageNY curriculum, which is designed to be more developmentally appropriate for the students.	

Slide 21
Time Allotted: 8 minutes
	Critical Point
	• The EngageNY Module Overview contains important information to orient teachers to the flow of lessons across the module. It also identifies focus standards, foundational standards from prior grades, and links to Standards for Mathematical Practice.
	
	Step-By-Step Instructions
	• Pass out handout **M4_P-05 ENY G6 Module 2 Overview.**
	• Say,
	  ○ "Recall from Module 3 that each module in EngageNY has a Module Overview that provides important information as you prepare to teach."
	  ○ "During today's session, you will experience a lesson from EngageNY that directly ties to the standards you explored in the vertical alignment discussion in Session 1. The lesson is a 6th grade lesson related to dividing fractions."
	• Give participants 1 minute to review the handout and the components of the Module Overview, listed on this slide.
	• Say,
	  ○ "As you scanned the Module Overview document, you should see a connection to the work we did in Session 1. Notice that this overview helps identify key content and practice standards. It also provides a look at foundational standards from prior grades. Some of these standards are on the vertical alignment study you completed in Session 1."
	  ○ "The EngageNY lesson you will experience today is the fourth of eight lessons about dividing fractions in Topic A of Module 2 in 6th grade. Please take 1
		M4_P-05 ENY G6 Module 2 Overview

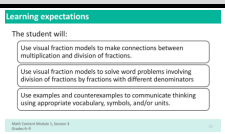
Time & Slide Thumbnail	Facilitator Notes Session 3	Materials

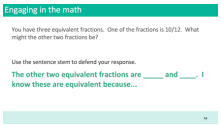
	minute to read the Module Overview to help situate the lesson in the flow of the entire module."
	• After participants read the Module Overview, call on several participants to share important takeaways from the Module Overview.
	  ○ Possible responses: The second paragraph of the module overview highlights important ideas developed in Topic A. Students explore real-life situations that require them to ask "How much is one share?" and "What part of the unit is one share?"
	  ○ Students are presented with conceptual problems where they determine the quotient represents how many of the divisor is in the dividend.
	  ○ Students look for and uncover patterns while modeling quotients of fractions to ultimately discover the relationship between multiplication and division. They use patterns to discover the algorithm for dividing fractions.
	
	Words of Wisdom
	• Stay focused on the connection to standards in the Module Overview.
	• Encourage participants to explore the rest of the Module Overview on their own time.	


Slide 22
Time Allotted: 2 minutes
	Critical Idea
	• The key shifts are what make the LSSM different from previous standards. These key shifts should guide decisions about teaching and learning.
	
	Step-by-Step Instructions
	• Say,
	  ○ "You are going to experience a mathematics lesson from EngageNY to give you an opportunity to observe how the key shifts come to life in the classroom while students engage in the mathematics content."
	  ○ "In order to help you with this experience, recall the tool we used in previous modules that will help you look for and document evidence of the key shifts."
	• Tell participants to locate handout **M4_P-06 Looking for Evidence of the Key Shifts**.
	• **<u>Animate the slide.</u>**
	• Direct participants to read the directions on the slide and then orient them to the handout. Say,
	  ○ "The left-hand column contains descriptors of the key shifts. Next to each descriptor is a box that can be used to check off any of the student actions that you observe in the lesson.
	  ○ "The right-hand column should be used to document observations of specific actions that can be used as evidence for the descriptors that have been checked off in the left-hand column.
	• Ensure there are no additional questions.
	
	Words of Wisdom
	• Make sure all participants have an opportunity to read through the document to familiarize themselves with the descriptors of the key shifts. They will use these descriptors to make observations while they experience the math lesson.
		M4_P-06 Looking for Evidence of the Key Shifts

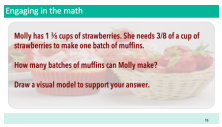
	Critical Idea	

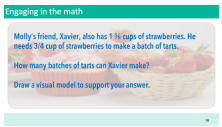
Time & Slide Thumbnail	Facilitator Notes Session 3	Materials


Slide 23
Time Allotted: 2 minutes
	• This slide clarifies the learning expectations of the lesson and opens by connecting to prior knowledge required for the lesson.
	
	Step-By-Step Instructions
	• Say,
	  ○ "Session 3 is about experiencing the lesson as a student. After, you will have a chance to unpack the lesson and make connections to the standards."
	  ○ "The lesson is a 6th grade lesson related to dividing fractions. Although it may be tempting to discuss the lesson or talk about instructional changes to consider, I encourage you to stay in the learner role so you can get the most out of the experience."
	  ○ "Today's lesson will bring together your work with operations with rational numbers up to this point."
	• Review the learning expectations for the day on the slide.
	  ○ Use visual fraction models to make connections between multiplication and division of fractions.
	  ○ Solve word problems involving division of fractions by fractions with different denominators.
	  ○ Use examples and counterexamples to communicate thinking using appropriate vocabulary, symbols, and/or units.
	
	Words of Wisdom
	• The third learning expectation relates to SMP 3 and SMP 6. This could surface again on Slide 30 when participants share evidence of student engagement in the key shifts, specifically in building conceptual understanding and procedural skills and fluency.	

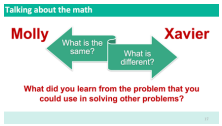
 Slide 24 Time Allotted: 7 minutes	***Critical Idea*** • Participants take on the student perspective to experience an EngageNY lesson related to the big idea studied in Session 1. ***Step-By-Step Instructions*** • Direct participants to handout **M4_P-07 ENY G6 M2, TA, L4 Student**. This is the student version of the EngageNY lesson. • **<u>Animate the slide</u>**. Read the prompt on the screen and give participants 2 minutes to respond on handout **M4_P-07.** • **<u>Animate the slide</u>**. Direct participants to turn and talk with a partner using the sentence stem on screen. Circulate around the room while participants are discussing and make note of any common or unique strategies that you hear. • Ask 1 or 2 volunteers to share their responses. • Say, "In today's lesson, you will apply what you know about equivalent fractions to solve problems." ***Words of Wisdom*** • By allowing participants to choose equivalent fractions, the problem becomes more accessible to all participants. By discussing their reasoning using the sentence stem, each participant will be exposed to different strategies for finding equivalent fractions.	Facilitators should study and prepare to teach the EngageNY lesson before the session using handout **M4_P-08** ENY G6 M2, TA, L4 Teacher. **M4_P-07 ENY G6 M2, TA, L4 Student**

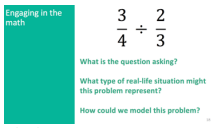
Time & Slide Thumbnail	Facilitator Notes Session 3	Materials


Slide 25
Time Allotted: 10 minutes
	Critical Idea
	• This example is a review of the problems completed in the previous EngageNY lesson in Module 2 to divide fractions with the same denominator.
	
	Step-By-Step Instructions
	• Give participants a few minutes of quiet time to individually work the problem displayed on the slide. Participants should show work on handout **M4_P-07 ENY G6 M2, TA, L4 Student** to justify their reasoning.
	• Circulate around the room while participants are working, finding a few different methods of solving the problem.
	• Debrief the task by asking participants the following questions:
	  ○ What given information was important? Why?
	  ○ What was required for you to answer the question?
	• Ask 2 or 3 participants to share their solutions and methods for solving the problem with the whole group, and connect it back to their responses in the whole-group debrief.
	  ○ Possible responses:
	  ▪ Molly can make 3 and 2/3 batches of muffins. [11/8 divided by 3/8]
	  ▪ Molly can make almost 4 batches of muffins. [Approximating the answer].
	
	Words of Wisdom
	• Most participants will have the same method of solving the problem. If participants finish the task quickly, challenge them to think of an alternate approach to solving.
		M4_P-07 ENY G6 M2, TA, L4 Student
		
		Optional: Virtual manipulatives
		
		Fraction bars: https://www.mathplayground.com/Fraction_bars.html
		
		Image credit: aizram18/iStock/ Thinkstock


Slide 26
Time Allotted: 10 minutes
	Critical Idea
	• This example extends beyond the problems completed in the previous EngageNY lesson and requires participants to divide fractions with unlike denominators.
	
	Step-By-Step Instructions
	• Give participants a few minutes of quiet time to individually work the problem displayed on the slide. Participants should show work on handout **M4_P-07 ENY G6 M2, TA, L4 Student** to justify their reasoning.
	• Just as you did on the previous problem, circulate around the room while participants are working, finding a few different methods of solving the problem.
	• Debrief the task by asking participants the same questions as before:
	  ○ What given information was important? Why?
	  ○ What was required for you to answer the question?
	• Ask 2 or 3 participants to share their solutions and methods for solving the problem with the whole group.
	  ○ Possible responses:
	  ▪ Xavier can make 1 and 5/6 batches of muffins. [11/8 divided by 3/4]
	  ▪ Xavier can make almost 2 batches of muffins. [Approximating the answer].
	
	Words of Wisdom
	• While participants are working, listen closely for the strategies used to solve the problem. If most of the participants are solving using the same method,
		M4_P-07 ENY G6 M2, TA, L4 Student
		
		Image credit: aizram18/iStock/ Thinkstock

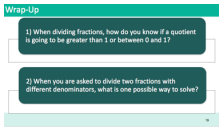
Time & Slide Thumbnail	Facilitator Notes Session 3	Materials
	encourage them to justify their reasoning using a visual model and then compare their visual model to a partner's and answer, "How are the models the same? How are they different?"	

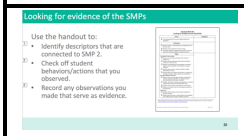
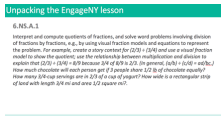

Slide 27
Time Allotted: 5 minutes
	Critical Idea
	• Intentional teacher moves support focus, coherence, and rigor around the mathematics content.
	
	Step-By-Step Instructions
	• Ask participants to quietly think about the similarities and differences between the two problems they just solved. Give 1 minute to discuss with a partner.
	• Call on 2 volunteers to share one idea they discussed with their partner. Possible responses:
	  ○ In Molly's situation, the given information included fractions with common denominators. In Xavier's situation, the given information included fractions with unlike denominators.
	  ○ In both situations, you had to determine how many batches the person could make by dividing the total amount by the fractional part required to make each batch. To solve the problem, one must determine how many shares are in the total amount.
	  ○ Fraction bars and number lines are useful to visualize the shares and the total amount being divided.
	  ○ One method of solving could be to find common denominators between the fractions being compared, and then divide across (numerators by numerators, denominators by denominators).
	
	Words of Wisdom
	• This slide provides a break in the lesson to allow for student discourse about the first two problems they solved and to compare the similarities and differences in dividing fractions with like and unlike denominators.
	• Regardless of the method used to solve the problem, participants should discover patterns to dividing fractions by fractions that eventually lead to the development of the algorithm, rather than being given an algorithm and then told to use to solve the division problems.	

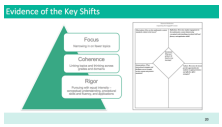
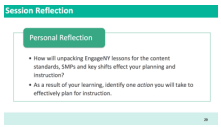

Slide 28
Time Allotted: 20 minutes
	Critical Idea
	• Intentional teacher moves support focus, coherence, and rigor around the mathematics content.
	
	Step-By-Step Instructions
	• Direct participants to the problem on the screen. Before participants solve the problem, ask the following questions. Say,
	  ○ "What is this question asking?" [It could be either 2/3 of what is ¾ or how many 2/3 are in ¾?]
	  ○ "What type of real-life situation might this problem represent?" [How many 2/3 pieces of pie are in ¾ of a pie? I can do 2/3 of a job in ¾ of an hour. How much can I do in an hour?]
	  ○ "Is this problem similar to Molly's or Xavier's situation? Why?" [The fractions in this problem do not have common denominators, so the problem is similar to Xavier's situation.]
		M4_P-07 ENY G6 M2, TA, L4 Student

Time & Slide Thumbnail	Facilitator Notes Session 3	Materials

	• After the whole-group discussion, give participants 5 minutes to solve the problem and use a visual model to justify their solution. While participants are working, circulate around the room and look for different strategies used to solve the problem. Make note of specific strategies to share with the whole group.
	• Once participants are done solving, debrief the problem with the whole group. Ask the following questions:
	  ○ "Is your answer larger or smaller than one? Why?" [Since 2/3 is less than ¾, we will have an answer that is larger than 1.]
	  ○ "How did you approach solving the problem?" [Some participants may use equivalent fractions with like denominators to model and divide, such as $\frac{9}{12} \div \frac{8}{12}$. 9 units divided by 8 units = $\frac{9}{8}$ or $1\frac{1}{8}$ units.]
	• After the whole group debrief, direct participants to work in pairs or alone to solve Exercises 1-5 on handout **M4_P-07 ENY G6 M2, TA, L4 Student**. Participants are no longer required to draw models; however, participants can choose to create a visual to complete the problems.
	
	Words of Wisdom
	• Before facilitating, it is important to anticipate student responses. Be sure to work through the teacher lesson plan and sample problems ahead of time. While participants are working on the problem, circulate around the room to see if the strategies used are similar to the strategies you anticipated.	


Slide 29
Time Allotted: 10 minutes
	Critical Idea
	• Reflecting on learning is important to solidifying understanding of the mathematics content.
	
	Step-By-Step Instructions
	• Direct participants to independently respond to the reflection questions on handout **M4_P-00 Module 4 Presentation Slides**.
	• After participants have responded on their own paper, direct them to stand up and form triads with two other people from different tables and count off A-B-C.
	• Say,
	  ○ "Now you will take turns sharing your reflections with your small group. Each of you will have the opportunity to respond to both questions, and each round will take 2 minutes."
	  ○ "When time begins, person A will have 1 minute to share their thoughts about question 1. After 1 minute, the other two group members can respond to what person A shared by asking clarifying questions, restating what was said, or extending ideas shared."
	  ○ "Then repeat the process with the second question."
	  ○ "You will have a total of 4 minutes to discuss with your triad."
	• Ensure there are no questions and then direct the group to begin their discussion. Circulate around the room while triads are discussing and listen for any important takeaways to share with the whole group.
	• After 4 minutes, debrief with the whole group. Share any important takeaways that you heard in the small group discussions. If it has not surfaced already, highlight the following:
	  ○ When the dividend is larger than the divisor, the quotient will be larger than 1.
		M4_P-00 Module 4 Presentation Slides

Time & Slide Thumbnail	Facilitator Notes Session 3	Materials
	○ It is possible to get a whole number quotient when dividing fractions. ○ To divide fractions with different denominators, one strategy to solving is to find equivalent fractions with like denominators. ***Words of Wisdom*** • Circulate around the room while triads are discussing. Listen for any important takeaways to share with the whole group.	
 Slide 30 Time Allotted: 10 minutes	***Critical Idea*** • Intentional teacher moves support focus, coherence, and rigor around the mathematics content. ***Step-By-Step Instructions*** • Say, "It is essential that we take a deliberate pause in our work to reflect on this experience. During the lesson, you were asked to use handout **M4_P-06 Looking for Evidence of Student Engagement in the Key Shifts** to check off student behaviors/actions observed in the lesson. Please take 3 minutes of independent think time to record specific evidence of the descriptors in the right-hand column of the page." • Once participants are done, say, "Please look over the evidence you just recorded on handout M4_P-04. Which key shift did you engage in the most?" • Direct the participants to find someone in the room they have not worked with in this session and share their responses. • After all pairs have the opportunity to share, call on 2 or 3 participants to summarize their discussions. • Summarize the learning experience. Say, "Now that you have experienced an EngageNY lesson from start to finish and made connections between the learning experience and the key shifts of the LSSM, we will unpack the structure of the EngageNY lesson and discuss implications for planning and instruction. " ***Words of Wisdom*** • Facilitator should designate each corner of the room as A-B-C-D ahead of time. • Don't rush this conversation. Participants need time to think about, reflect on, and share their observations of what they observed in the lesson.	**M4_P-06 Looking for Evidence of Student Engagement in the Key Shifts**
 Slide 31 Time Allotted: 7 minutes	***Critical Idea*** • A strong understanding of the LSSM is critical for teachers to implement the Tier 1 resources with fidelity and integrity. Teachers must be able to use the language of the standards to guide high-quality, standards-aligned teaching and learning. ***Step-By-Step Instructions*** • Direct participants to handout **M4_P-08 ENY G6 M2, TA, L4 Teacher**. • **<u>Animate the slide.</u>** Display the content standard addressed by this lesson. • Say, ○ "When planning with any resource, it is important that we consider how the lesson, task, or activity aligns with the standards." ○ "Only when we truly understand how the standards manifest in the lesson can we ensure that our instruction will align with the full intent of the standards."	**M4_P-08 ENY G6 M2, TA, L4 Teacher**

Time & Slide Thumbnail	Facilitator Notes Session 3	Materials
	○ "On the slide, you will find the standard this lesson was intended to address." • **<u>Animate the slide</u>**. Give participants a few moments to read the full text of the standard on the slide. • Direct participants to reflect on the math learning experience. Say, "For the next several minutes, I would like for your table groups to discuss the lesson and look for evidence of how this lesson meets the intent of the content and practice standards identified in the lesson." • Allow 5 minutes for the groups to look for and discuss evidence of the standards in the lesson, using handout M4_P-07. • When time is up, ask for a representative from a few table groups to share. Probe for specific evidence demonstrated in the lesson that meets the standards. • Transition to the next slide by saying, "Not all lessons address every aspect of a standard. Often, we will find that a lesson covers only a piece of the standard. This is why it is important to consider not only the individual lesson, but also how that lesson fits into the full scope and sequence. The Module Overview document can help to make those connections. Only then can we tell if a standard is covered to its full intent." ***Words of Wisdom*** • The EngageNY teacher lesson plans identify both content and practice standards for each lesson.	
 Slide 32 Time Allotted: 23 minutes	***Critical Idea*** • Participants should be able to identify evidence of each of the three key shifts in the EngageNY lesson. ***Step-By-Step Instructions*** • Direct participants to the following handouts: **M4_P-08 ENY G6 M2, TA, L4 Teacher** and **M4_P-09 Diamond Reflection: Unpacking the ENY Lesson**. • Say, ○ "With your table groups, unpack the lesson using handout M4_P-08. Discuss the questions and record your responses on the paper." ○ "Be prepared to share specific places in the lesson that you see each of the key shifts being addressed." ○ "Remember, when looking for evidence of rigor, you will first need to discuss which aspect of rigor (conceptual understanding, procedural skills and fluency, or application) is addressed with this standard." • Give participants time to analyze and discuss the lesson and to look for evidence of the key shifts. • Have 2 or 3 groups share what they discussed as evidence for each shift. ***Words of Wisdom*** • Circulate around the room while participants are working.	**M4_P-08 ENY G6 M2, TA, L4 Teacher** **M4_P-09 Diamond Reflection: Unpacking the ENY Lesson**
 	***Critical Idea*** • Reflecting on our learning is important to solidifying understanding and making progress toward accomplishing the goals of the initiative. ***Step-By-Step Instructions*** • Give participants 3 minutes to reflect on the questions. Participants should record	**M4_P-00 Module 4 Presentation Slides**

Time & Slide Thumbnail	Facilitator Notes Session 3	Materials

Slide 33
Time Allotted: 5 minutes
	their responses on handout **M4_P-00 Module 4 Presentation Slides** for the slide that matches the screen.
	• Once participants have completed their reflections, call on 2 or 3 participants to share their responses. Then close the session. Say,
	  ○ "The goals of Session 2 were to experience an EngageNY lesson as a learner, identify evidence of the key shifts, and examine teacher resources related to the EngageNY lesson. "
	  ○ "Your experience as a learner and your collaborative conversations with colleagues in this session helped identify implications for teaching and learning."
	  ○ "In Session 3, we will use tools for purposeful planning to align teaching and learning of the EngageNY curriculum to the intent of the standards and to meet the needs of all students."
	• If participants have any lingering questions after the session, direct them to place the questions on the Parking Lot poster before they leave the session. If necessary, address the questions in Session 2.
	
	Words of Wisdom
	• This is an opportunity for participants to reflect on and share their learning from Session 3.	

Math Module 4, Session 4 Overview

Session Title: Purposeful Planning of the EngageNY Curriculum

Grade Band: 6–9

Session Overview *What are the overarching goals of this module? Why are those goals important? How do they connect to what's come before and what will come after?*
In Session 4, participants review and apply the process for collaborative planning conversations to connect foundational study of the standards to the EngageNY resource materials. In this session, participants use the Planning Guide along with the Louisiana Teacher Resources to study standards and analyze EngageNY lessons for an upcoming unit of study. In Modules 5 and 6, participants will continue to use the Planning Guide to plan lessons for an upcoming unit of study.

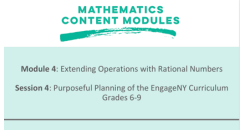
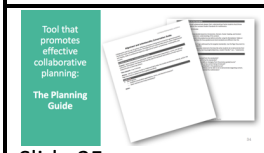
Session Objectives *What will participants know, believe, and be able to do as a result of this session?*	**Session Products** *What will participants produce during the session? Immediately following the session?*
Participants will be able to… • Use the Louisiana Teacher Resources to investigate standards related to an upcoming unit of study. (CK, CS) • Develop and use a common understanding of grade-level standards to align teaching and learning. (CK, CS) CK: Content Knowledge CP: Content Pedagogy CS: Curriculum Skills	• Completed Planning Guide for standards and EngageNY lessons related to an upcoming unit of study
Session Key Points *What are the 2-4* most *important takeaways from this session?*	**Session Agenda at a Glance** *What is the overall flow of this session?*

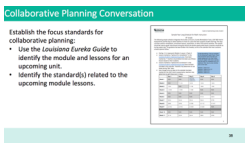
Participants will know…
• Collaborative planning is:
  ○ Working and learning together to plan instruction (including lessons, units, assessments, and activities) focused on building the intended learnings described by the standards.
  ○ Discussing, interpreting, and refining curriculum resource materials together in order use them to best meet our students' needs in their learning.
• Focusing planning conversations around the standards in order to make informed instructional decisions will:
  ○ Build a shared understanding of the math content described in the standards and what that content looks like in teaching and learning.
  ○ Consider and use instructional strategies and resources that will best build on students' current understanding and connect to the intended learning.
	• Do a quick review:
	  ○ Planning Guide process
	  ○ Foundational Study of the Standards
	  ○ Bridge to Lesson Planning
	• Do independent practice:
	  ○ Complete a Planning Guide Conversation to plan for an upcoming unit of study.
	• Reflect on purposeful planning.

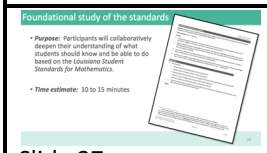
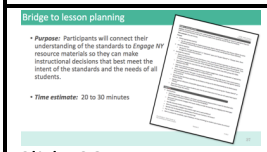
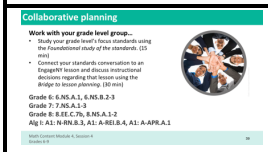
Facilitation Recommendations	
Session Length	60 minutes
Time of Year	

Materials	
Session Materials: • Handout **M4_P-10 Planning Guide** **Additional Supplies:** ● N/A	**Facilitator Pre-Work and Prep:** • Print participant handouts (1 per participant). • Ensure internet access is available. • Practice navigating through the Louisiana Believes K-12 planning resources website: ○ Louisiana Guide to Implementing Eureka ○ Remediation Guide ○ Companion Document ○ Louisiana Student Standards for Mathematics

Participant Pre-Work	
Participants must bring: • Laptop or device with internet capability	**Participants:** ● N/A

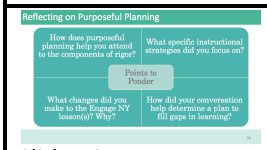
Time & Slide Thumbnail	Facilitator Notes Session 4	Materials
 Slide 34 Time Allotted: < 1 minute	**Session 4: Purposeful Planning of the EngageNY Curriculum** **Slides 34–40: 60 minutes** ***Critical Idea*** • This slide is an introductory slide for Session 4. • We will work toward implementing structured collaboration for planning instruction. ***Step-by-Step Instructions*** • Tell participants that we will transition into Session 4. ***Words of Wisdom*** • For Session 4, participants should be sitting in grade-level groups.	
 Slide 35 Time Allotted: 3 minutes	***Critical Idea*** • This tool is helpful for grade level teams to stay focused on studying standards and planning aligned instruction together. ***Step-By-Step Instructions*** • Say, "In the previous modules, we introduced you to the Planning Guide which is a tool used to establish a regular, ongoing collaborative planning process that will help us to plan instruction in our grade level teams that is aligned to the standards." • Give each participant a copy of **M4_P-10 Planning Guide**. • Say, "Remember that this tool is designed to be completed within a 45-minute collaborative planning period. This tool guides the discussion, keeping the team focused on understanding the content and what it looks like in instruction, and then considering the best use of strategies and resources for instruction." • Give participants 2 minutes with a partner to re-familiarize themselves with the Planning Guide. ***Words of Wisdom*** • This is a quick re-introduction to the Planning Guide. Do not take too much time here.	**M4_P-10 Planning Guide**


Slide 36
Time Allotted: 2 minutes
	Critical Idea
	• This tool is helpful for grade level teams to stay focused on studying standards and planning aligned instruction together.
	
	Step-By-Step Instructions
	• Say,
	  ○ "If you recall, the first step in the process is to choose focus standards to study. When you are using the tool with your team, you might choose standards that are coming up in the next lesson or unit, you might choose standards that the team typically sees students struggle with . . . But whatever way you decide, make sure the team agrees to generally 1–3 standards to study."
	  ○ "Today, you will use the Planning Guide to guide your planning for an upcoming module. Use the Louisiana Eureka Guide for your grade level to identify the module and lessons for an upcoming unit of study. Once you have identified the module and lesson, identify the standard(s) related to the
		M4_P-10 Planning Guide
		
		Internet access so participants can access the Louisiana Guide to Implementing Eureka for their grade-level or course: https://www.louisianabelieves.com/resources/library/k-12-

Time & Slide Thumbnail	Facilitator Notes Session 4	Materials
	upcoming module lessons. These standards will serve as the focus for collaborative planning today." • Have each group take note of the standards they will study today on the first page of their Planning Guide. ***Words of Wisdom*** • Walk around to groups throughout this process to monitor progress, and ask questions/prompt groups to maintain progress.	math-year-long-planning
 Slide 37 Time Allotted: 2 minutes	***Critical Idea*** • This tool is helpful for grade level teams to stay focused on studying standards and planning aligned instruction together. ***Step-By-Step Instructions*** • Say, ○ "Since I guided you through this process step by step in the last module, this time I am going to quickly review the process and then release you to do it on your own in your grade level groups." ○ "If you look at the next page of the Planning Guide, you will see the section called Foundational Study of the Standards." ○ "Remember, the purpose of this section is to collaboratively deepen your understanding of what students should know and be able to do based on the standards and that this section should only take about 15 minutes to complete, once you get used to the process." ○ "There is a set of specific 'look-fors' listed on your page to help guide you in this conversation." ***Words of Wisdom*** • These slides serve as a quick review of the Planning Guide.	**M4_P-10 Planning Guide** Laptop with internet access
 Slide 38 Time Allotted: 3 minutes	***Critical Idea*** • This tool is helpful for grade level teams to stay focused on studying standards and planning aligned instruction together. ***Step-By-Step Instructions*** • Say, ○ "Locate the third page of the Planning Guide. This section is called the Bridge to Lesson Planning. We did a deep dive into this section in the last module." ○ "Remember, the purpose of this section is to connect your understanding of the standards to the EngageNY resource materials." ○ "You will recall that this part of the collaborative planning conversation takes the most time, about 20 to 30 minutes." ***Words of Wisdom*** • These slides serve as a quick review of the Planning Guide.	**M4_P-10 Planning Guide**
	***Critical Idea*** • It is important to apply a deep study of the standards to the implications for instruction to make informed decisions about resource materials that are used in classrooms.	**M4_P-10 Planning Guide** Laptops with internet access

Time & Slide Thumbnail	Facilitator Notes Session 4	Materials

Slide 39
Time Allotted: 45 minutes
	Step-By-Step Instructions
	• Say,
	  ○ "You will now work in grade-level groups to work through the entire planning process laid out in the Planning Guide. Use the grade-level standards you discussed in Session 1 to guide your discussion. These standards are displayed on the screen."
	  ○ "You will have 45 minutes to complete the whole process, 15 minutes for the study of the standards, and 30 minutes for the Bridge to Lesson Planning."
	  ○ "Remember to use all resources available to you on the Louisiana Believes site to help you with your discussion and to help you in choosing a lesson from EngageNY."
	  ○ "I will be walking around the entire time to make myself available if you need any help or have any questions."
	• Allow 45 minutes for participants to plan.
	
	Words of Wisdom
	• Circulate around the room and assist groups as needed.
		Image credit: Creatas Images/Creatas/Thinkstock

 Slide 40 Time Allotted: 5 minutes	***Critical Idea*** • It is important to apply our deep study of the standards to the implications for instruction to help us make informed decisions about the resource materials that we use in our classrooms. ***Step-By-Step Instructions*** • Say, "Take a few moments to read the questions on the slide and individually reflect on a sticky note." • Give participants a minute to individually reflect and write down their thinking on a sticky note. • Give them a few minutes to discuss the questions with those in their group. • After 3 minutes, ask participants to share out a few of their conversation points. • Summarize by reminding participants that it is our job as teachers to provide students with rich learning experiences that give them opportunities to engage in meaningful mathematics and that help prepare them for college and career. • Ask participants to return to their original seats. ***Words of Wisdom*** • Don't rush this conversation—it is pivotal.	Sticky notes

Math Module 4 Closing Slides

Grade Band: 6–9

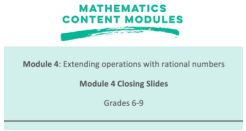
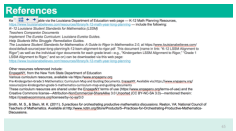
Session Overview *What are the overarching goals of this session? Why are those goals important? How do they connect to what's come before and what will come after?*	
Participants reflect on their learning in Module 4 and complete a participant evaluation survey.	
Session Key Points *What are the 2-4* most *important takeaways from this session?*	**Session Products** *What will participants produce during the session? Immediately following the session?*
Participants will know… • Content to be covered in Modules 5 and 6.	• Complete session evaluation survey

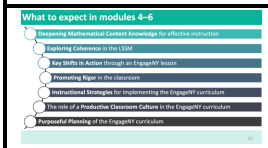
Session Objectives *What will participants know, believe, and be able to do as a result of this session?*	**Session Agenda at a Glance** *What is the overall flow of this session?*
Participants will be able to... • Summarize learning from Module 4.	• Quick review: What to expect in Modules 5 and 6 • Module 4 participant evaluation survey

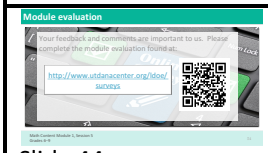
Facilitation Recommendations	
Session Length	15 minutes

Preparing to Facilitate *What materials and supplies are needed to facilitate this session? What must the facilitator do to prep for the actual session?*	
Session Materials: • Computer or device with internet access **Additional Supplies:** • N/A	**Facilitator Pre-Work and Prep:** • N/A

Participant Pre-Work *What should participants bring to this session? What should they do or prepare in advance of this session?*	
Participants must bring: • Computer or device with internet access	**Before this session, participants should:** • Experience Module 4.

Time & Slide Thumbnail	Facilitator Notes Module 4 Closing Slides	Materials
 Slide 41 Time Allotted: < 1 minute	**Module 4 Closing Slides: Reflecting on Learning** **Slides 43–47: 15 minutes** ***Critical Idea*** • This slide is an introductory slide for the Module 4 closing slides. ***Step-by-Step Instructions*** • Say, "We will conclude Module 4 and preview what is to come in Modules 5 and 6." ***Words of Wisdom*** • Participants should have a laptop or device with internet capability to complete the participant evaluation survey.	
 Slide 42 Time Allotted: < 1 minute	***Critical Idea*** • This slide provides references for Module 4. ***Step-By-Step Instructions*** • Say, "These are the references for Module 4." ***Words of Wisdom*** • This slide should be brief.	


Slide 43
Time Allotted: < 1 minute
	Critical Idea
	• This slide provides participants with an overview of the content in Modules 4–6.
	
	Step-By-Step Instructions
	• Say, "Before we conclude Module 4, we will remind you of the content and structure of Modules 5 and 6. Just as in Modules 1–4, the learning sessions in future modules will include mathematics tasks and discussions designed to strengthen your understanding of the LSSM to support students' learning of mathematics content through the high-quality curriculum."
	• **<u>Animate the slide one bullet at a time.</u>**
	• As each bullet animates, say, "In each module, we will…
	  ○ Analyze the major work of grades 6–9 to **Deepen Mathematical Content Knowledge** for effective instruction.
	  ○ **Explore Coherence** between and within the grade-level standards in the LSSM.
	  ○ See the **Key Shifts in Action** through an EngageNY lesson related to the major work of grades 6–9**.**
	  ○ Study the Standards for Mathematical Practice to **Promote Rigor** in the classroom.
	  ○ Identify the role of a **Productive Classroom Culture** in the EngageNY curriculum.
	  ○ Apply knowledge of the LSSM to guide **Purposeful Planning** of the EngageNY curriculum."
	• Summarize the slide. Say, "Please keep at the forefront of your mind that the work in which you are engaged is part of a bigger picture, and this first module serves as the foundation for learning in Modules 5 and 6."
	• Thank the participants for their time and effort in Module 4.	

Time & Slide Thumbnail	Facilitator Notes Module 4 Closing Slides	Materials
	Words of Wisdom • This slide should be brief and is about clarifying the learning expectations of Modules 5 and 6. It is a review from Modules 1–3.	
 Slide 44 Time Allotted: 12 minutes	***Critical Idea*** • Participant feedback is highly valued and is critical in informing future learning sessions. ***Step-By-Step Instructions*** • Say, "In order to gather feedback of your learning experience, you will take a module evaluation survey. Your responses are very important, so you will use the next 10 minutes to complete the survey." ***Words of Wisdom*** • Participants should complete the evaluation survey during the session.	http://www.utdanacenter.org/ldoe/surveys