Table of Contents[1]

Arithmetic Operations Including Division of Fractions

Module Overview 3

Topic A: Dividing Fractions by Fractions (**6.NS.A.1**) 9

Lesson 1: Interpreting Division of a Fraction by a Whole Number—Visual Models 10

Lesson 2: Interpreting Division of a Whole Number by a Fraction—Visual Models 22

Lessons 3–4: Interpreting and Computing Division of a Fraction by a Fraction—More Models 33

Lesson 5: Creating Division Stories 55

Lesson 6: More Division Stories 65

Lesson 7: The Relationship Between Visual Fraction Models and Equations 73

Lesson 8: Dividing Fractions and Mixed Numbers 82

Topic B: Multi-Digit Decimal Operations—Adding, Subtracting, and Multiplying (**6.NS.B.3**) 88

Lesson 9: Sums and Differences of Decimals 89

Lesson 10: The Distributive Property and the Products of Decimals 95

Lesson 11: Fraction Multiplication and the Products of Decimals 100

Mid-Module Assessment and Rubric 105
Topics A through B (assessment 1 day, return 1 day, remediation or further applications 1 day)

Topic C: Dividing Whole Numbers and Decimals (**6.NS.B.2**, **6.NS.B.3**) 117

Lesson 12: Estimating Digits in a Quotient 118

Lesson 13: Dividing Multi-Digit Numbers Using the Algorithm 131

Lesson 14: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Fractions 142

Lesson 15: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Mental Math 156

[1]Each lesson is ONE day, and ONE day is considered a 45-minute period.

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Topic D: Number Theory—Thinking Logically About Multiplicative Arithmetic (**6.NS.B.4**) 164

Lesson 16: Even and Odd Numbers 165

Lesson 17: Divisibility Tests for 3 and 9 173

Lesson 18: Least Common Multiple and Greatest Common Factor 182

Lesson 19: The Euclidean Algorithm as an Application of the Long Division Algorithm 197

End-of-Module Assessment and Rubric 206
Topics A through D (assessment 1 day, return 1 day, remediation or further applications 1 day)

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Grade 6 • Module 2

Arithmetic Operations Including Division of Fractions

OVERVIEW

In Module 1, students used their existing understanding of multiplication and division as they began their study of ratios and rates. In Module 2, students complete their understanding of the four operations as they study division of whole numbers, division by a fraction, and operations on multi-digit decimals. This expanded understanding serves to complete their study of the four operations with positive rational numbers, thereby preparing students for understanding, locating, and ordering negative rational numbers (Module 3) and algebraic expressions (Module 4).

In Topic A, students extend their previous understanding of multiplication and division to divide fractions by fractions. They construct division stories and solve word problems involving division of fractions (**6.NS.A.1**). Through the context of word problems, students understand and use partitive division of fractions to determine how much is in each group. They explore real-life situations that require them to ask, "How much is one share?" and "What part of the unit is that share?" Students use measurement to determine quotients of fractions. They are presented conceptual problems where they determine that the quotient represents how many of the divisor is in the dividend. For example, students understand that $\frac{6\text{ cm}}{2\text{ cm}}$ derives a quotient of 3 because 2 divides into 6 three times. They apply this method to quotients of fractions, understanding $\frac{6}{7} \div \frac{2}{7} = \frac{6\text{ sevenths}}{2\text{ sevenths}} = 3$ because, again, 2 divides into 6 three times. Students look for and uncover patterns while modeling quotients of fractions to ultimately discover the relationship between multiplication and division. Using this relationship, students create equations and formulas to represent and solve problems. Later in the module, students learn the direct correlation of division of fractions to division of decimals along with the application of this concept.

Prior to division of decimals, students revisit all decimal operations in Topic B. Students have had extensive experience with decimal operations to the hundredths and thousandths (**5.NBT.B.7**), which prepares them to easily compute with more decimal places. Students begin by relating the first lesson in this topic to the last lesson in Topic A, which focused on mixed numbers. They find that sums and differences of large mixed numbers can sometimes be more efficiently determined by first converting the number to a decimal and then applying the standard algorithms (**6.NS.B.3**). They use estimation to justify their answers.

Within decimal multiplication, students begin to practice the distributive property. Students use arrays and partial products to understand and apply the distributive property as they solve multiplication problems involving decimals. By gaining fluency in the distributive property throughout this module and the next, students become proficient in applying the distributive property in Module 4 (**6.EE.A.3**). Estimation and place

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value enable students to determine the placement of the decimal point in products and recognize that the size of a product is relative to each factor. Students learn to use connections between fraction multiplication and decimal multiplication.

In Grades 4 and 5, students used concrete models, pictorial representations, and properties to divide whole numbers (**4.NBT.B.6**, **5.NBT.B.6**). They became efficient in applying the standard algorithm for long division. Students broke dividends apart into like base ten units, applying the distributive property to find quotients place by place. In Topic C, students connect estimation to place value and determine that the standard algorithm is simply a tally system arranged in place value columns (**6.NS.B.2**). Students understand that when they "bring down" the next digit in the algorithm, they are essentially distributing, recording, and shifting to the next place value. They understand that the steps in the algorithm continually provide better approximations to the answer. Students further their understanding of division as they develop fluency in the use of the standard algorithm to divide multi-digit decimals (**6.NS.B.3**). They make connections to division of fractions and rely on mental math strategies to implement the division algorithm when finding the quotients of decimals.

In the final topic, students think logically about multiplicative arithmetic. In Topic D, students apply odd and even number properties and divisibility rules to find factors and multiples. They extend this application to consider common factors and multiples and find greatest common factors and least common multiples. Students explore and discover that Euclid's algorithm is a more efficient way to find the greatest common factor of larger numbers and see that Euclid's algorithm is based on long division.

The module comprises 19 lessons; six days are reserved for administering the Mid- and End-of-Module Assessments, returning the assessments, and remediating or providing further applications of the concepts. The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

Focus Standards

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for* $(2/3) \div (3/4)$ *and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that* $(2/3) \div (3/4) = 8/9$ *because* $3/4$ *of* $8/9$ *is* $2/3$. *(In general,* $(a/b) \div (c/d) = ad/bc$.*) How much chocolate will each person get if* 3 *people share* $1/2$ lb *of chocolate equally? How many* $3/4$*-cup servings are in* $2/3$ *of a cup of yogurt? How wide is a rectangular strip of land with length* $3/4$ *mi and area* $1/2$ *square mi?*

Compute fluently with multi-digit numbers and find common factors and multiples.

6.NS.B.2 Fluently divide multi-digit numbers using the standard algorithm.

6.NS.B.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

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6.NS.B.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express* $36 + 8$ *as* $4\ (9 + 2)$.

Foundational Standards

Gain familiarity with factors and multiples.

4.OA.B.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Understand the place value system.

5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.B.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show* $(2/3) \times 4 = 8/3$, *and create a story context for this equation. Do the same with* $(2/3) \times (4/5) = 8/15$. *(In general,* $(a/b) \times (c/d) = ac/bd$.*)*

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5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by fractions.[2]

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for* $(1/3) \div 4$*, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $(1/3) \div 4 = 1/12$ *because* $(1/12) \times 4 = 1/3$.

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for* $4 \div (1/5)$*, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $4 \div (1/5) = 20$ *because* $20 \times (1/5) = 4$.

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students use concrete representations when understanding the meaning of division and apply it to the division of fractions. They ask themselves, "What is this problem asking me to find?" For instance, when determining the quotient of fractions, students ask themselves how many sets or groups of the divisor are in the dividend. That quantity is the quotient of the problem. They solve simpler problems to gain insight into the solution. They confirm, for example, that $10 \div 2$ can be found by determining how many groups of two are in ten. They apply that strategy to the division of fractions. Students may use pictorial representations such as area models, array models, number lines, and drawings to conceptualize and solve problems.

MP.2 **Reason abstractly and quantitatively.** Students make sense of quantities and their relationships in problems. They understand "how many" as it pertains to the divisor in a quotient of fractions problem. They understand and use connections between divisibility and the greatest common factor to apply the distributive property. Students consider units and labels for numbers in contextual problems and consistently refer to what the labels represent to make sense in the problem. Students rely on estimation and properties of operations to justify the reason for their answers when manipulating decimal numbers and their operations. Students reason abstractly when applying place value and fraction sense when determining the placement of a decimal point.

MP.6 **Attend to precision.** Students use precise language and place value when adding, subtracting, multiplying, and dividing by multi-digit decimal numbers. Students read decimal numbers using place value. For example, 326.31 is read as three hundred twenty-six and thirty-one hundredths. Students calculate sums, differences, products, and quotients of decimal numbers with a degree of precision appropriate to the problem context.

[2]Students who are able to multiply fractions in general can develop strategies to divide fractions in general by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement in Grade 5.

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MP.7 **Look for and make use of structure**. Students find patterns and connections when multiplying and dividing multi-digit decimals. For instance, they use place value to recognize that the quotient of $22.5 \div 0.15$ is the same as the quotient of $2250 \div 15$. In the example $36 + 48 = 12(3 + 4)$, students recognize that when expressing the sum of two whole numbers using the distributive property, the number 12 represents the greatest common factor of 36 and 48 and that 36 and 48 are both multiples of 12. When dividing fractions, students recognize and make use of a related multiplication problem or create a number line and use skip-counting to determine the number of times the divisor is added to obtain the dividend. Students use the familiar structure of long division to find the greatest common factor in another way, specifically the Euclidean algorithm.

MP.8 **Look for and express regularity in repeated reasoning**. Students determine reasonable answers to problems involving operations with decimals. Estimation skills and compatible numbers are used. For instance, when 24.385 is divided by 3.91, students determine that the answer is close to the quotient of $24 \div 4$, which equals 6. Students discover, relate, and apply strategies when problem solving, such as the use of the distributive property to solve a multiplication problem involving fractions and/or decimals (e.g., $350 \times 1.8 = 350(1 + 0.8) = 350 + 280 = 630$). When dividing fractions, students may use the following reasoning: Since $\frac{2}{7} + \frac{2}{7} + \frac{2}{7} = \frac{6}{7}$, then $\frac{6}{7} \div \frac{2}{7} = 3$, so I can solve fraction division problems by first getting common denominators and then solving the division problem created by the numerators. Students understand the long division algorithm and the continual breakdown of the dividend into different place value units. Further, students use those repeated calculations and reasoning to determine the greatest common factor of two numbers using the Euclidean algorithm.

Terminology

New or Recently Introduced Terms

- **Greatest Common Factor** (The *greatest common factor* of two whole numbers (not both zero) is the greatest whole number that is a factor of each number.

  For example, the GCF of 24 and 36 is 12 because when all of the whole number factors of 24 and 36 are listed, the largest factor they share is 12.)
- **Least Common Multiple** (The *least common multiple* of two whole numbers is the smallest whole number greater than zero that is a multiple of each number.

  For example, the LCM of 4 and 6 is 12 because when the multiples of 4 and 6 are listed, the smallest or first multiple they share is 12.)
- **Multiplicative Inverses** (A *multiplicative inverse of a number* is a number such that the product of both numbers is 1.

  For example, $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses of one another because $\frac{3}{4} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} = 1$.

  Multiplicative inverses do not always have to be the reciprocal. For example, $\frac{1}{5}$ and $\frac{10}{2}$ both have a product of 1, which makes them multiplicative inverses of each other.)

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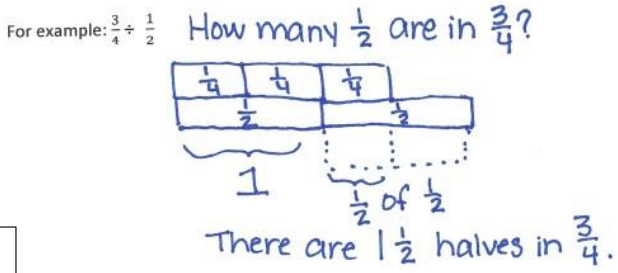
Familiar Terms and Symbols[3]

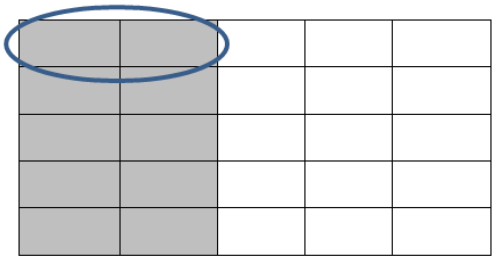
- Algorithm
- Composite Number
- Distributive Property
- Dividend
- Divisor
- Estimate
- Factors
- Multiples
- Prime Number
- Reciprocal

Suggested Tools and Representations

- Counters
- Fraction Tiles (example shown to the right)
- Tape Diagrams
- Area Models (example shown below)



$\frac{2}{5} \div 5$



Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	6.NS.A.1, 6.NS.B.3
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	6.NS.A.1, 6.NS.B.2, 6.NS.B.3, 6.NS.B.4

[3]These are terms and symbols students have seen previously.

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