MATHEMATICS CONTENT MODULES

Module 2: Multiplicative Thinking: Ratios and Rates
Session 0: Setting the Stage for Module 2

Grades 6–9

The goal of the initiative

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Improve the quality of instruction in math classrooms by developing deep mathematical content knowledge of teachers and connecting that content knowledge to the practical implementation of a quality curriculum.

Math Content Module 2
Grades 6–9

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Module 2 goals

Participants will

- Describe the vertical articulation of standards for the big idea: Using multiplicative thinking to reason about ratios and rates.
- Understand how EngageNY resources can be used to support the Louisiana Student Standards for Mathematics content and practice standards and how the practice standards support the key shifts in instruction.
- Closely examine Modeling with Mathematics and recognize its critical role in promoting rigor in the classroom.
- Examine research—and identify clear action steps—to foster persistence and effective effort in the classroom.

Math Content Module 2
Grades 6–9

3

Group norms

- Understand that those who work, learn.
- Look for solutions, not blame.
- Focus on systems, not people.
- Recognize that everyone has expertise.
- Be honest.
- Challenge ideas.
- Share talk time.
- Mistakes are expected, respected, and inspected.

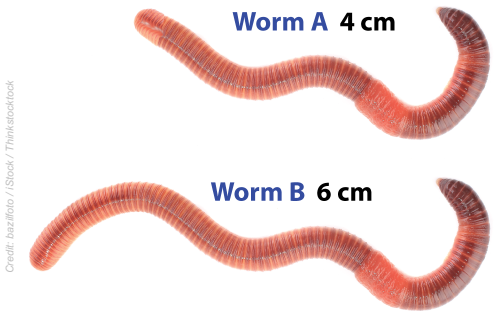
MATHEMATICS CONTENT MODULES

Module 2: Multiplicative Thinking: Ratios and Rates

Session 1: Deepening Mathematical Content Knowledge for Effective Instruction

Grades 6–9

Defining the big idea: Using multiplicative thinking to reason about ratios and rates



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Why is it important that students learn to reason multiplicatively in middle school?

Create a poster that captures your groups' understanding of how multiplicative thinking manifests itself at your grade level.

Math Content Module 2, Session 1
Grades 6–9

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Defining the big idea: Gallery walk

Review each of the posters from grade 6 through Algebra I. As you review the posters, consider the following:

- What do you notice about the posters from each grade level?
- Do you see any common ideas across the grade levels?
- Do you see evidence of the big idea developing from grade to grade?
- Do you see any gaps between grade levels?

Exploring tasks

Handout M2 P-02
Grade 6 – Algebra I Standards

Grade/ Course	Louisiana State Standards for Mathematics
Alg I	A1: S-ID.C.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8	8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
7	7.RP.A.2: Recognize and represent proportional relationships between quantities. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*
6	6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

As a table group, review each of the four tasks on your table. Discuss the mathematics involved in each task. Then order the tasks from grade 6 to Algebra I.

- What is the math required in each task?
- How does the required math for each task progress?

Use the standards to check the order of the four tasks.

Exploring tasks

Reflection

How did looking at the standards verify the order of the tasks?

What evidence of multiplicative thinking did you see in these tasks?

Why is it important to study standards across grade levels?

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MATHEMATICS CONTENT MODULES

Module 2: Multiplicative Thinking: Ratios and Rates

Session 2: Exploring Coherence in the LSSM

Grades 6–9

Exploring vertical alignment

Why study the vertical alignment of standards?

- Helps us understand our grade-level expectations
- Gives us information on student expectations from previous grades so that we can connect to prior learning and remediate, if necessary
- Gives us information on where students will be going with the material
- Gives us an idea of how the math develops and of our role within the larger system
- Provides insights on the type of instruction and experiences we need to provide our students

Vertical alignment

Using multiplicative thinking to reason with ratios and rates

Use ratio and rate reasoning to solve real-world and mathematical problem
diagrams, double number line diagrams, or equations.
Make tables of equivalent ratios rela
that can be modeled with linear functions and with exponential functions.
equal differences over equal intervals, and that exponential functions grow by equal factors

Interpret multiplication as scaling (resizing), by:
d. Relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world problems (e.g., convert 5 cm to 0.05 m; 9 ft to 108 in).

constant rate per unit interval relative to another.
the context of the data.
ems involving

Multiply or divide to solve word pro
for the unknown number to represe
as many vs. 6 more than).[1]
Explain why a fraction a/b
Interpret the slope (rate o
rate as the slope of the graph.
Graph proportional relationships, inter
represented in different ways. *For example, compare a distance-time graph to a distance-time e*
moving objects has greater speed.
Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordi
plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.
Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...

ratios in a table or graphing on a
verbal descriptions of proportional
to the number n of items purchased
expressed as t = pn.
ituation, with special attention to the
such as computing actual lengths and areas from a scale drawing and

Vertical alignment

Vertical Alignment Chart

Big Idea: Using Multiplicative Thinking to Reason About Ratios and Rates

Grade/ Course	LSS	Content Standards Summary
Alg I	A1: F-LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
	A1: S-ID.C.7	Interpret the slope (rate of change) ~~and the intercept (constant term)~~ of a linear model in the context of the data.
8	8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
	8.EE.B.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.
7	7.RP.A.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
	7.G.A.1	Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Vertical alignment

6	6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what unit rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
	6.EE.C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*
5	5.NF.B.5	Interpret multiplication as scaling (resizing), by: d. Relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
	5.MD.A.1	Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world problems (e.g., convert 5 cm to 0.05 m; 9 ft to 108 in).
4	4.OA.A.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and/or equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison (Example: 6 times as many vs. 6 more than).[1]
	4.NF.A.1	Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)
	4.MD.A.1	Know relative sizes of measurement units within one system of units including ft, in; km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. (Conversions are limited to one-step conversions.) *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

Vertical alignment discussion questions

- What content changes occur from grade to grade?
- Where are concepts introduced, developed, and finalized?
- Does an idea or skill get more complex, and if so, how?

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Have your thoughts about the big idea changed after studying the standards?

What are the implications for you as you plan instruction?

How does studying the standards in this way make evident the 3 key shifts of focus, coherence, and rigor?

MATHEMATICS CONTENT MODULES

Module 2: Multiplicative Thinking: Ratios and Rates

Session 3: Key Shifts in Action Through an EngageNY Lesson

Grades 6–9

EngageNY module overview

- Overview
- Focus Standards
- Foundational Standards

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Evidence of student engagement in the key shifts

Use the handout to:

- Check off student behaviors/actions that you observed.
- Record any observations you made that serve as evidence.

Looking for Evidence of Student Engagement in the Key Shifts

Focus	Evidence
☐ The learning goal(s) of the lesson supports grade level standard(s).	
Coherence	
☐ The lesson intentionally relates new concepts to students' prior skills and knowledge. ☐ Students set the foundation for future learning. ☐ Students access prior learning from major work in the grade in order to support new learning.	
Rigor	
Conceptual Understanding ☐ Students access concepts and ideas from a variety of perspectives. ☐ Students explain mathematical ideas behind a particular concept in a variety of ways. ☐ Students use examples and counterexamples to make and support conjectures applied to one problem to multiple situations. ☐ Students create and use a variety of models to analyze relationships. ☐ Students make use of patterns and structure to compose and decompose numbers, shapes, expressions, and equations. **Procedural Skills and Fluency** ☐ Students select tools (e.g. physical objects, manipulatives, drawings, diagrams, algorithms, or strategies) that are relevant and useful for the task or problem. ☐ Students communicate thinking using appropriate vocabulary, symbols and/or units in precise and accurate ways. ☐ Students look for patterns, generalizations, and shortcuts. ☐ Students are flexible in their use of procedures and skills to solve problems. **Application** ☐ Students decontextualize and contextualize quantities in problem situations. ☐ Students plan and choose a solution pathway when applying their mathematical knowledge to different situations.	

Note: To help educators look for evidence of grade-level-appropriate student engagement in mathematical tasks, these narrative descriptors are adapted from Illustrative Mathematics. (2014, February 12). *Standards for Mathematical Practice: Commentary and Elaborations for K-5 and 6-8*. Tucson, AZ. Available at http://commoncoretools.me/2014/02/12/k-5-elaborations-of-the-practice-standards

Learning expectations

The student will:

- Examine situations carefully to decide whether two quantities are proportional to one another.
- Compare and contrast proportional and non-proportional relationships.
- Use examples and counterexamples to communicate thinking using appropriate vocabulary, symbols, and/or units.

Reviewing proportional relationships

Think-Pair-Share

- What does it mean for two quantities to be proportional to one another?
- What methods do you have in your toolbox for determining if a relationship is proportional?

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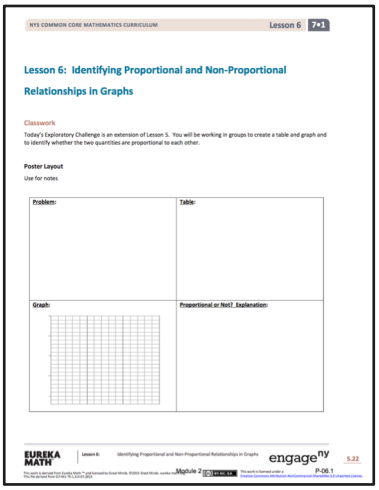
Identifying proportional and non-proportional relationships


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 6 7•1

Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs

Classwork

Today's Exploratory Challenge is an extension of Lesson 5. You will be working in groups to create a table and graph and to identify whether the two quantities are proportional to each other.

Poster Layout

Use for notes

Problem:	Table:
Graph:	Proportional or Not? Explanation:

EUREKA MATH | Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs engage^ny S.22

Module 2 P-06.1

With your group,

- Create a table and graph to determine whether the given quantities are proportional to one another.
- Then create a poster using the same layout on your handout.

Math Content Module 2, Session 3
Grades 6–9

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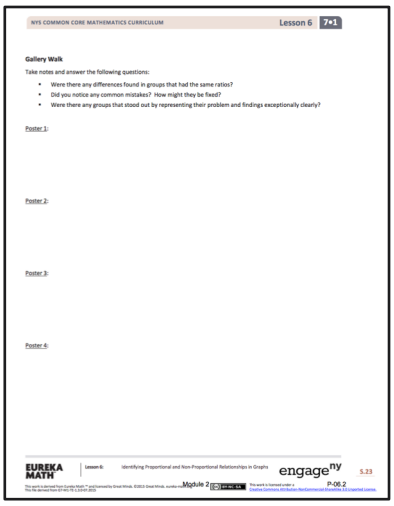
Gallery walk

Make note of the following:

- Similarities and differences between posters
- Common mistakes
- Clear justifications for identifying a proportional or non-proportional situation



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 6 7•1

Gallery Walk

Take notes and answer the following questions:

- Were there any differences found in groups that had the same ratios?
- Did you notice any common mistakes? How might they be fixed?
- Were there any groups that stood out by representing their problem and findings exceptionally clearly?

Poster 1:

Poster 2:

Poster 3:

Poster 4:

EUREKA MATH | Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs engage^ny S.23

Module 2 P-06.2

Math Content Module 2, Session 3
Grades 6–9

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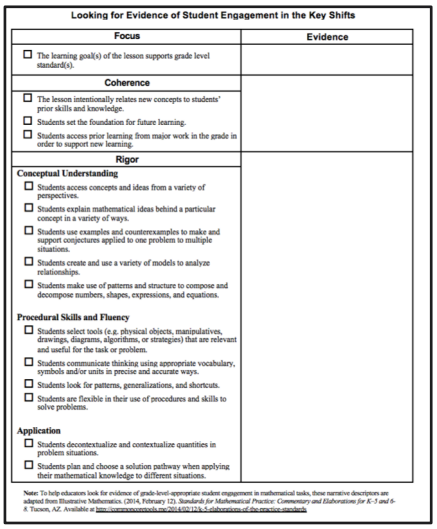

Looking for Evidence of Student Engagement in the Key Shifts

Focus	Evidence
☐ The learning goal(s) of the lesson supports grade level standard(s).	
Coherence	
☐ The lesson intentionally relates new concepts to students' prior skills and knowledge. ☐ Students set the foundation for future learning. ☐ Students access prior learning from major work in the grade in order to support new learning.	
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Grades 6–9

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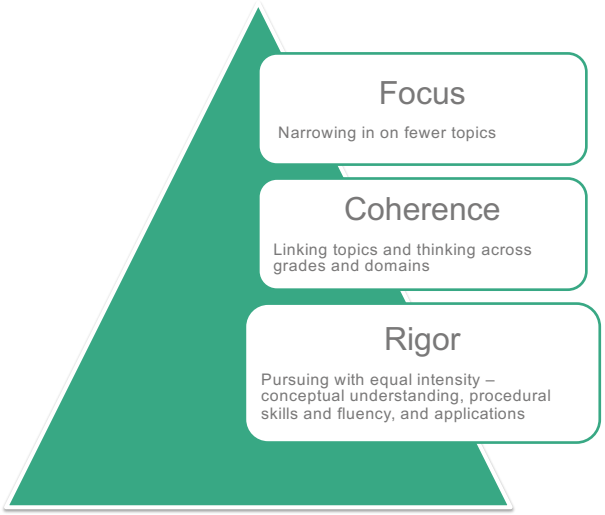
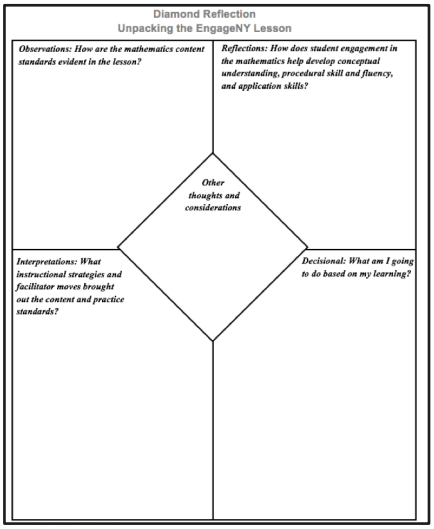
Unpacking the EngageNY lesson

7.RP.A.2a

Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Evidence of the key shifts





Unpacking the EngageNY lesson

- What tasks or activities would you do—in grade 6, grade 8, and grade 9—that either support or extend the work done in this grade 7 lesson?
- How did the process of studying the vertical alignment of the standards enrich your discussion of this 7th-grade lesson and connections to other grade levels?

Math Content Module 2, Session 3
Grades 6–9

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MATHEMATICS CONTENT MODULES

Module 2: Multiplicative Thinking: Ratios and Rates

Session 4: Instructional Strategies to Improve Curriculum Implementation

Grades 6–9

Standards for Mathematical Practice K–12

The Louisiana Student Standards for Mathematics consist of two corresponding and connected sets of standards:

1. Standards for Mathematical Practice
   - Apply to all grade levels
   - Describe mathematically proficient students

2. Standards for Mathematical Content
   - K–8 standards presented by grade level
   - High school standards presented by high school course

Math Content Module 2, Session 4
Grades 6–9

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Exploring modeling with mathematics: Chalk talk

- What is modeling with mathematics?
- What is NOT modeling with mathematics?
- How and why is modeling with mathematics useful?
- What structures need to be in place for students to engage in modeling with mathematics?

Math Content Module 2, Session 4
Grades 6–9

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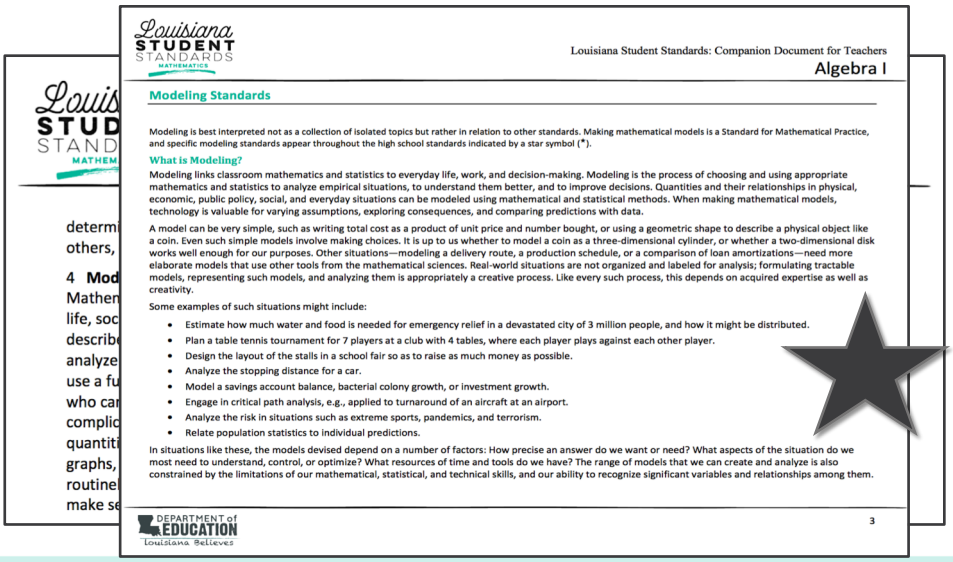
Exploring modeling with mathematics: LSSM



Louisiana STUDENT STANDARDS MATHEMATICS

Louisiana Student Standards: Companion Document for Teachers
Algebra I

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

What is Modeling?

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimate how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Plan a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Design the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyze the stopping distance for a car.
- Model a savings account balance, bacterial colony growth, or investment growth.
- Engage in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyze the risk in situations such as extreme sports, pandemics, and terrorism.
- Relate population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them.

DEPARTMENT of EDUCATION Louisiana Believes

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Math Content Module 2, Session 4
Grades 6–9

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Exploring modeling with mathematics: LSSM

The modeling cycle involves:

1) Identifying essential variables in a situation
2) Formulating models and creating a representation that describes the relationship between the variables
3) Performing operations using those models and drawing conclusions
4) Interpreting the results of those operations in context
5) Validating the conclusions of those results and improving the model if necessary
6) Reporting on the conclusions and the reasoning behind them

Choices, assumptions and approximations are present throughout this cycle.

Math Content Module 2, Session 4
Grades 6–9

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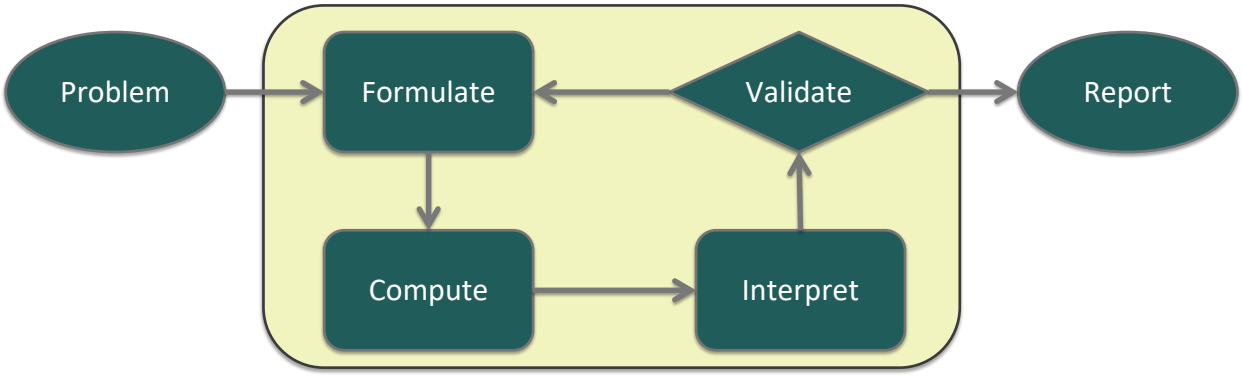
Exploring modeling with mathematics: LSSM



Exploring modeling with mathematics: Act one

Source: http://www.101qs.com/2714-super-stairs

Exploring modeling with mathematics: Act three

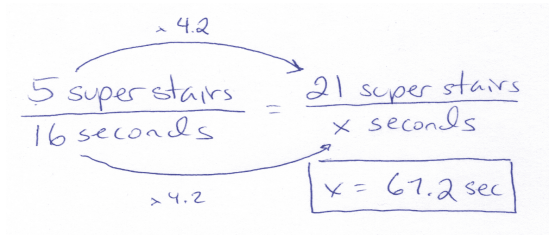
Source: http://www.101qs.com/2714-super-stairs

Exploring modeling with mathematics: The sequels

Option 1:

If someone wanted to run 1,000,000 steps, what kind of super staircase would that require?

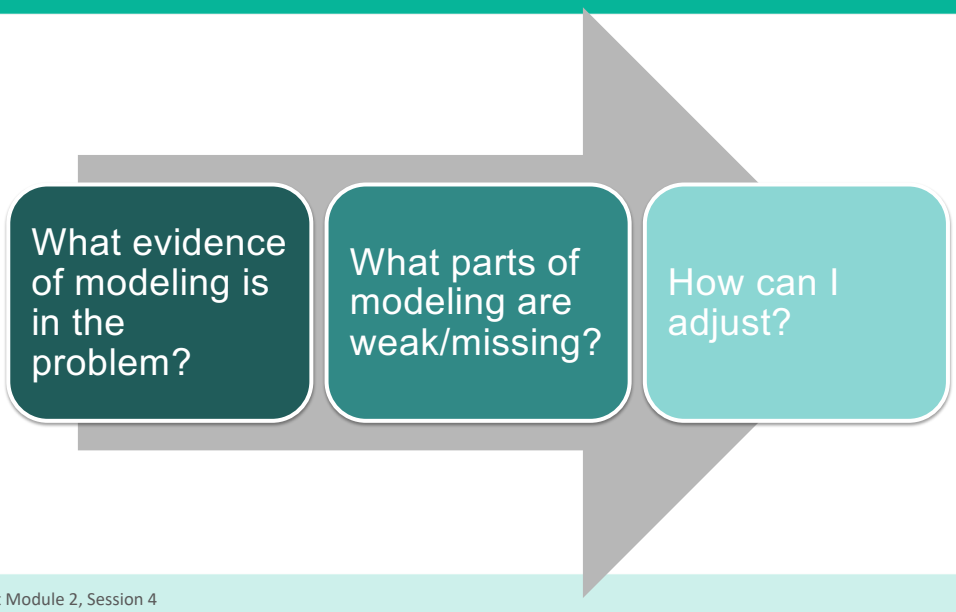
Option 2:



The above student found the answer to the original question a different way.
Is she right? Why or why not?

Source: http://www.101qs.com/2714-super-stairs

Exploring modeling with mathematics: Evidence



Math Content Module 2, Session 4
Grades 6–9

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Exploring modeling with mathematics: Evidence

Sentence stem:
We saw ***evidence*** of modeling with mathematics in our task when … but felt as if we could enhance it by…

Image credit: drogatnev / iStock / Thinkstock

Math Content Module 2, Session 4
Grades 6–9

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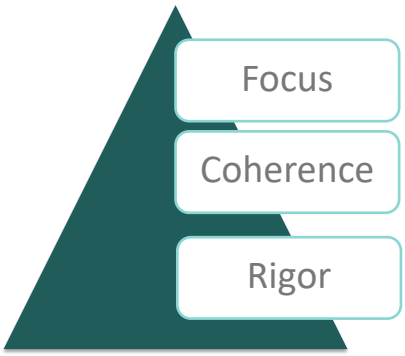
Exploring modeling with mathematics: Key shifts

How does intentional integration of modeling with mathematics foster rigor in the classroom?



Narrowing in on fewer topics

Linking topics and thinking across grades

Pursuing—with equal intensity—conceptual understanding, procedural skills and fluency, and applications

Math Content Module 2, Session 4
Grades 6–9

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MATHEMATICS CONTENT MODULES

Module 2: Multiplicative Thinking: Ratios and Rates

Session 5: The Role of a Productive Classroom Culture in the EngageNY Curriculum

Grades 6–9

Consider a time when you struggled to learn something new in either a personal or professional situation.

- Describe the situation.
- What behaviors contributed to overcoming the struggle?

Persistence and effective effort

Read the Agile Mind article, *Persistence and effective effort.*

Highlight a **word**, a **phrase**, and a **sentence** that is important for your work with students.

Persistence and effective effort
Use with *Exploring* "Key Influences on learning and achievement"

Persistence is continuation of effort and striving in the face of difficulty, opposition, or failure: it is a key characteristic of successful people across professional and academic disciplines. Persistence is evidenced by willingness to continue to try in the face of challenge. For students, this persistence can be a driving force to help them achieve their academic and personal goals.

But where does persistence in the face of adversity come from, and how can educators and parents promote it? Persistence, or **academic resilience**—academic achievement despite a challenging or threatening circumstance in the educational process—is often described as an outcome of high motivation and positive self-beliefs. For example, students with a strong belief in their own capabilities are shown to persist longer through academic difficulty.

Adults play a significant role in helping students develop persistence and apply **effective effort**—proven strategies paired with attitudes and beliefs that enable students to see the greatest benefit. They can reinforce a view of intelligence as malleable, or changeable, provide frequent and specific feedback to students on their academic progress, and encourage students to reflect on their own experiences with overcoming challenges and succeeding.

Educators can foster in students specific skills that are associated with effective effort, such as time management, organizational strategies, and goal setting. When students have a plan for how to deal with challenging academic situations that includes metacognitive strategies, or strategies for examining their own thinking, they are better equipped to continue to try when they struggle.

And classroom practice and routines influence students' effort. Students are more likely to

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An Educator's Course in Academic Youth Development

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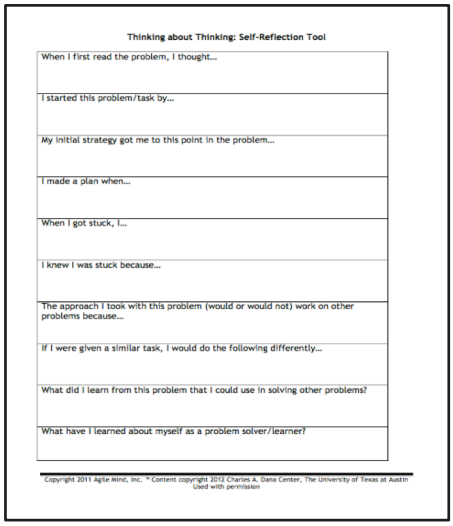
Persistence and effective effort



Thinking about Thinking: Self-Reflection Tool

When I first read the problem, I thought...

I started this problem/task by...

My initial strategy got me to this point in the problem...

I made a plan when...

When I got stuck, I...

I knew I was stuck because...

The approach I took with this problem (would or would not) work on other problems because...

If I were given a similar task, I would do the following differently...

What did I learn from this problem that I could use in solving other problems?

What have I learned about myself as a problem solver/learner?

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Reflect on the Super Stairs activity from Session 4, using the *Thinking about Thinking: Self-Reflection Tool.*

How could a tool like this be helpful for your students? With colleagues in your district?

Persistence and effective effort

Key ideas from research

People become smarter with time, effective effort, and persistence on challenging tasks.

Planning, monitoring, and evaluating are metacognitive strategies that increase learning.

Educator actions

Provide students with challenging academic tasks and effectively facilitate student persistence through struggle.

Model for students that mistakes are part of learning and encourage them to take academic risks.

Teach and reinforce the use of metacognitive strategies in a variety of problem situations.

Provide feedback to students on using and developing problem-solving and personal strategies.

Persistence and effective effort

Identify places in the lesson where you could support students in persistence and effective effort, including:

- Places where students are likely to struggle
- Places where you can model that mistakes and taking risks are how we learn
- Places where you can provide feedback to students that encourages metacognition and other effective-effort strategies
- Places where you can apply the specific educator actions recorded during the discussion on the previous slide.

Share your learning!

- Identify 1 way where you can model persistence through a think-aloud or similar strategy.
- Identify 1 way you will anticipate students' struggle. Prepare appropriate responses.

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Persistence and effective effort

What? What is one important takeaway from your learning?

So What? In what ways can you increase opportunities for students to take intellectual risks in your classroom routines?

Now What? What actions will you take as a result of this learning?

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MATHEMATICS CONTENT MODULES

Module 2: Multiplicative Thinking: Ratios and Rates

Module 2 Closing Slides and Participant Evaluation Survey

Grades 6–9

References

Key resources available via the Louisiana Department of Education web page — K-12 Math Planning Resources, https://www.louisianabelieves.com/resources/library/k-12-math-year-long-planning — include the following:

- *K–12 Louisiana Student Standards for Mathematics (LSSM)*
- *Teachers Companion Documents*
- *Implement The Eureka Curriculum: Louisiana Eureka Guides.*
- *Help Students Who Struggle: Remediation Guides*.
- *The Louisiana Student Standards for Mathematics: A Guide to Rigor in Mathematics 2.0*, at https://www.louisianabelieves.com/docs/default-source/year-long-planning/k-12-lssm-alignment-to-rigor.pdf
  This document (name in link: "K-12 LSSM Alignment to Rigor") as well as the individual rigor documents for each grade level—e.g., "Kindergarten LSSM Alignment to Rigor," "Grade 1 LSSM Alignment to Rigor," and so on) can be downloaded via this web page: https://www.louisianabelieves.com/resources/library/k-12-math-year-long-planning

Other resources include:
From education company Agile Mind, Inc.:

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- Used with permission of Agile Mind, Inc., "Thinking About Thinking: Self-Reflection Tool," which appears in Summer-Start Academic Youth Development, School-Year Academic Youth Development, and An Educator's Course in Academic Youth Development, by the Charles A. Dana Center and Agile Mind, Inc., available to users of Dana Center/Agile Mind programs in mathematics and science. To learn more about the Academic Youth Development family of programs, see http://www.agilemind.com/programs/academic-youth-development.

Dan Meyer. (2013 December and subsequent posts). Super Stairs, part of 101 Questions. Available at http://www.101qs.com/2714-super-stairs

From the New York State Department of Education: EngageNY
Various curriculum resources, available via https://www.engageny.org
These curriculum resources are shared under the EngageNY terms of use (https://www.engageny.org/terms-of-use) and the Creative Commons license—Attribution-NonCommercial-ShareAlike 3.0 Unported (CC BY-NC-SA 3.0)—mentioned therein: https://creativecommons.org/licenses/by-nc-sa/3.0

From the National School Reform Faculty (NSRF). (n.d.). Text Rendering Experience Available at https://www.nsrfharmony.org/system/files/protocols/text_rendering_0.pdf

What to expect in Modules 3, 4, 5, and 6

- **Deepening Mathematical Content Knowledge** for effective instruction
- **Exploring Coherence** in the LSSM
- **Key Shifts in Action** through an EngageNY lesson
- **Promoting Rigor** in the classroom
- **Instructional Strategies** for implementing the EngageNY curriculum
- The role of a **Productive Classroom Culture** in the EngageNY curriculum
- **Purposeful Planning** of the EngageNY curriculum

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Module evaluation

Your feedback and comments are important to us. Please complete the module evaluation found at:

http://www.utdanacenter.org/ldoe/surveys

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