## Algebra II Learning Acceleration Guidance

Learning acceleration will ensure students have the skills they need to equitably access and practice on-grade level content. This chart is a reference guide for teachers to help them more quickly identify the specific prerequisite and co-requisitestandards necessary for every Algebra Il standard. Students should spend the large majority of their time on the major work of the grade ( $\square$ ). Supporting work ( $\square$ ) and, where appropriate, additional work ( $\quad$ ) can engage students in the major work of the grade.

| Algebra II Standard | Previous Grade(s) Standards | Algebra II Standards Taught in Advance | Algebra II Standards Taught Concurrently |
| :---: | :---: | :---: | :---: |
| A2: N-RN.A. 1 <br> Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1 / 3}$ to be the cube root of 5 because we want $\left(5^{1 / 3}\right)^{3}=5^{(1 / 3) 3}$ to hold, so $\left(5^{1 / 3}\right)^{3}$ must equal 5. | 8.EE.A. 1 <br> Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^{2} \times 3^{-5}=3^{-3}=$ $1 / 3^{3}=1 / 27$. <br> 8.EE.A. 2 <br> Use square root and cube root symbols to represent solutions to equations of the form $x^{2}=p$ and $x^{3}=p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{ } 2$ is irrational. |  |  |
| A2: N-RN.A. 2 <br> Rewrite expressions involving radicals and rational exponents using the properties of exponents. |  | A2: N-RN.A. 1 <br> Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1 / 3}$ to be the cube root of 5 because we want $\left(5^{1 / 3}\right)^{3}=5^{(1 / 3 / 3}$ to hold, so $\left(5^{1 / 3}\right)^{3}$ must equal 5. |  |
| A2: N-Q.A. 2 <br> Define appropriate quantities for the purpose of descriptive modeling. | A1: N-Q.A. 1 <br> Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. <br> A1: N-Q.A. 2 <br> Define appropriate quantities for the purpose of descriptive modeling. |  |  |

A2: N-CN.A. 1
Know there is a complex number $i$ such that $i^{2}=-1$, and every complex number has the form $a+b i$ with $a$ and $b$ real.

## A2: N-CN.A. 2

Use the relation $i^{2}=-1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

## A2: N-CN.C. 7

Solve quadratic equations with real coefficients that have complex solutions.

## A2: A-SSE.A. 2

Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-\left(y^{2}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as $\left(x^{2}-y^{2}\right)\left(x^{2}+\right.$ $y^{2}$ ).
8.EE.A. 2
Use square root and cube root symbols to represent solutions to equations of the form $x^{2}=p$ and $x^{3}=\mathrm{p}$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{ } 2$ is irrational.

## 7.EE.A. 1

Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients to include multiple grouping symbols (e.g. parentheses, brackets, and braces).

## A1: A-SSE.A. 1

Interpret expressions that represent a quantity in terms of its context.
a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^{n}$ as the product of $P$ and a factor not depending on $P$.

## A1: A-SSE.A. 2

Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-\left(y^{2}\right)^{2}$, or see $2 x^{2}+8 x$ as $(2 x)(x)+2 x(4)$, thus recognizing it as a polynomial whose terms are products of monomials and the polynomial can be factored as $2 x(x+4)$.

## A2: N-CN.A. 2

Use the relation $i^{2}=-1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

## A2: N-CN.A. 1

Know there is a complex number $i$ such that $i^{2}=$ -1 , and every complex number has the form $a+$ bi with $a$ and $b$ real.

## A2: A-SSE.B. 3 <br> Choose and produce an equivalent form of an

 expression to reveal and explain properties of the quantity represented by the expression.c. Use the properties of exponents to transform expressions for exponential functions. For example, the expression $1.15^{t}$ can be rewritten as $\left(1.15^{1 / 12}\right)^{12 t} \approx$ $1.012^{12 t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is $15 \%$.

## A2: A-SSE.B. 4

Apply the formula for the sum of a finite geometric series (when the common ratio is not 1 ), and use the formula to solve problems. For example, calculate mortgage payments.

## A2: A-APR.B. 2

Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x-a$ is $p(a)$, so $p(a)=$ 0 if and only if $(x-a)$ is a factor of $p(x)$.

A1: A-SSE.B. 3 expression to reveal and explain properties of the quantity represented by the expression.
a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions for exponential functions emphasizing integer exponents. For example, the growth of bacteria can be modeled by either $f(t)=$ $3^{(t+2)}$ or $g(t)=9\left(3^{t}\right)$ because the expression $3^{(t+2)}$ can be rewritten as $\left(3^{t}\right)\left(3^{2}\right)=9\left(3^{t}\right)$.

## A1: A-SSE.B. 3

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
d. Factor a quadratic expression to reveal the zeros of the function it defines.
e. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
f. Use the properties of exponents to transform expressions for exponential functions emphasizing integer exponents. For example, the growth of bacteria can be modeled by either $f(t)=$ $3^{(t+2)}$ or $g(t)=9\left(3^{t}\right)$ because the expression $3^{(t+2)}$ can be rewritten as $\left(3^{t}\right)\left(3^{2}\right)=9\left(3^{t}\right)$.

## A2: A-SSE.A. 2

Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-\left(y^{2}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as $\left(x^{2}-y^{2}\right)\left(x^{2}+\right.$ $y^{2}$ ).

## A2: A-SSE.B. 3

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
c. Use the properties of exponents to
transform expressions for exponential
functions. For example the expression
$1.15^{t}$ can be rewritten as $\left(1.15^{1 / 12}\right)^{12 t} \approx$
$1.012^{12 t}$ to reveal the approximate
equivalent monthly interest rate if the
annual rate is $15 \%$

## A2: A-APR.B. 3

Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

## A2: A-APR.D. 6

Rewrite simple rational expressions in different forms; write ${ }^{a(x)} / b(x)$ in the form $q(x)+{ }^{r(x)} / b(x)$, where $a(x), b(x), q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

## A2: A-APR.B. 3 <br> Identify zeros of polynomials when suitable

 factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
## A2: A-APR.C. 4

Use polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $\left(x^{2}+y^{2}\right)^{2}=$ $\left(x^{2}-y^{2}\right)^{2}+(2 x y)^{2}$ can be used to generate Pythagorean triples.

A1: A-SSE.B. 3
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions for exponential functions emphasizing integer exponents. For example, the growth of bacteria can be modeled by either $f(t)=$ $3^{(t+2)}$ or $g(t)=9\left(3^{t}\right)$ because the expression $3^{(t+2)}$ can be rewritten as $\left(3^{t}\right)\left(3^{2}\right)=9\left(3^{t}\right)$.

## A1: A-APR.B. 3

Identify zeros of quadratic functions, and use the zeros to sketch a graph of the function defined by the polynomial.

## A2: A-SSE.A. 2

Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-\left(y^{2}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as $\left(x^{2}-y^{2}\right)\left(x^{2}+\right.$ $y^{2}$ ).

## A2: A-SSE.A. 2

Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-\left(y^{2}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as $\left(x^{2}-y^{2}\right)\left(x^{2}+\right.$ $y^{2}$ ).

| Algebra II Standard | Previous Grade(s) Standards | Algebra Il Standards Taught in Advance | Algebra II Standards Taught Concurrently |
| :---: | :---: | :---: | :---: |
| A2: A-APR.D. 6 <br> Rewrite simple rational expressions in different forms; write $a(x) / b(x)$ in the form $q(x)$ $+r(x) / b(x)$, where $a(x), b(x), q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. | 7.NS.A. 2 <br> Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. <br> a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (1) $(-1)=1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing realworld contexts. <br> b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If $p$ and $q$ are integers, then $-(p / q)=(-$ $p) / q=p /(-q)$. Interpret quotients of rational numbers by describing realworld contexts. <br> c. Apply properties of operations as strategies to multiply and divide rational numbers. <br> d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. | A2: A-SSE.A. 2 <br> Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-\left(y^{2}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as $\left(x^{2}-y^{2}\right)\left(x^{2}+\right.$ $y^{2}$ ). | A2: A-APR.B. 2 <br> Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x-a$ is $p(a)$, so $p(a)=0$ if and only if $(x-a)$ is a factor of $p(x)$. |

## A2: A-CED.A. 1

Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

## A2: A-REI.A. 1

Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

## A2: A-REI.A. 2

Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A1: A-CED.A. 1
Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, and exponential functions.

## A1: A-REI.B. 4

Solve quadratic equations in one variable.
a. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x-p)^{2}=q$ that has the same solutions. Derive the quadratic formula from this form.
b. Solve quadratic equations by inspection (e.g., for $x^{2}=49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as "no real solution."

## A1: A-REI.A. 1

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

|  | A2: A-REI.A.1 <br> Explain each step in solving an equation as <br> following from the equality of numbers asserted <br> at the previous step, starting from the <br> assumption that the original equation has a <br> solution. Construct a viable argument to justify a <br> solution method. |
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|  |  |
| A2: A-REI.A.1 <br> Explain each step in solving an equation as <br> following from the equality of numbers <br> asserted at the previous step, starting from <br> the assumption that the original equation has <br> a solution. Construct a viable argument to <br> justify a solution method. | A2: A-CED.A.1 <br> Create equations and inequalities in one <br> variable and use them to solve problems. <br> Include equations arising from linear and <br> quadratic functions, and simple rational and <br> exponential functions. |

## A2: A-REI.B. 4

Solve quadratic equations in one variable.
b. Solve quadratic equations by inspection (e.g., for $x^{2}=49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm b i$ for real numbers $a$ and $b$.

## A2: A-REI.C. 6

Solve systems of linear equations exactly and approximately (e.g., with graphs), limited to systems of at most three equations and three variables. With graphic solutions, systems are limited to two variables.

## A2: A-REI.C. 7

Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y=-3 x$ and the circle $x^{2}+y^{2}=$ 3.

A1: A-REI.B. 4
Solve quadratic equations in one variable.
a. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x-p)^{2}=q$ that has the same solutions. Derive the quadratic formula from this form.
b. Solve quadratic equations by inspection (e.g., for $x^{2}=49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as "no real solution."

## A1: A-REI.C. 6

Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A2: A-REI.D. 11
Explain why the $x$-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

## A2: F-IF.B. 4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

## A1: A-REI.D. 11

Explain why the $x$-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, piecewise linear (to include absolute value), and exponential functions.

## A1: N-Q.A. 1

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

## A1: F-IF.A. 1

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y=f(x)$.

## A1: F-IF.B. 4

For linear, piecewise linear (to include absolute value), quadratic, and exponential functions that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

## A2: A-REI.C. 6

Solve systems of linear equations exactly and approximately (e.g., with graphs), limited to systems of at most three equations and three variables. With graphic solutions, systems are limited to two variables.

## A2: A-REI.C. 7

Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y=-3 x$ and the circle $x^{2}+y^{2}=3$.

## A2: F-BF.A. 1

Write a function that describes a relationship between two quantities.
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

## A2: F-IF.C. 7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

## A2: F-IF.B. 6

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

## A2: F-IF.C. 7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

A1: F-IF.A. 2
Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

## A1: F-IF.B. 6

Calculate and interpret the average rate of change of a linear, quadratic, piecewise linear (to include absolute value), and exponential function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

## A1: A-APR.B. 3

Identify zeros of quadratic functions, and use the zeros to sketch a graph of the function defined by the polynomial.

## A1: F-IF.A. 1

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y=f(x)$.

## A1: F-IF.C. 7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
b. Graph piecewise linear (to include absolute value) and exponential functions.

## A1: F-IF.C. 8

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

## A2: F-IF.B. 4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

## A2: F-IF.C. 8

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y=(1.02)^{t}, y=(0.97)^{t}, y=$ (1.01) $12^{\mathrm{t}}, \mathrm{y}=(1.2)^{\mathrm{t}} / 10$, and classify them as representing exponential growth or decay. A2: F-BF.B. 3 Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

A2: F-IF.C. 8
Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y=$ $(1.02)^{\mathrm{t}}, \mathrm{y}=(0.97)^{\mathrm{t}}, \mathrm{y}=(1.01) 12^{\mathrm{t}}, \mathrm{y}=$ $(1.2)^{t} / 10$, and classify them as representing exponential growth or decay.

## AZ: F-IF.C. 9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, determine which has the larger maximum.

A1: F-IF.C. 8
Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

## A1: F-IF.C. 9

Compare properties of two functions (linear, quadratic, piecewise linear [to include absolute value] or exponential) each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, determine which has the larger maximum.

## A2: N-RN.A. 1

Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1 / 3}$ to be the cube root of 5 because we want $\left(5^{1 / 3}\right)^{3}=5^{[1 / 3] 3}$ to hold, so $\left(5^{1 / 3}\right)^{3}$ must equal 5.

## A2: F-IF.B. 4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

## A2: F-BF.B. 3

Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

## A2: F-BF.A. 1

Write a function that describes a relationship between two quantities.
a. Determine an explicit expression, a recursive process, or steps for calculation from a context
b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

## A2: F-BF.A. 2

Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

A1: F-BF.A. 1 function that describes a relationship between two quantities.
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

## For a function that models a relationship

 between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
## A2: F-LE.A. 2

Given a graph, a description of a relationship, or two input-output pairs (include reading these from a table), construct linear and exponential functions, including arithmetic and geometric sequences to solve multistep problems.

## A2: F-LE.A. 2

Given a graph, a description of a relationship, or two input-output pairs (include reading these from a table), construct linear and exponential functions, including arithmetic and geometric sequences to solve multistep problems.

## A2: F-BF.B. 3

Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

A1: F-BF.B. 3
Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of $k$ (both positive and negative). Without technology, find the value of $k$ given the graphs of linear and quadratic functions. With technology, experiment with cases and illustrate an explanation of the effects on the graph that include cases where $f(x)$ is a linear, quadratic, piecewise linear (to include absolute value) or exponential function.

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
b. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

## A2: F-IF.C. 8

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y=$ $(1.02)^{\mathrm{t}}, \mathrm{y}=(0.97)^{\mathrm{t}}, \mathrm{y}=(1.01) 12^{\mathrm{t}}, \mathrm{y}=$ $(1.2)^{t} / 10$, and classify them as representing exponential growth or decay.

## A2: F-BF.B. 4

Find inverse functions.
a. Solve an equation of the form $f(x)=c$ for a simple function $f$ that has an inverse and write an expression for the inverse. For example, $f(x)=2 x^{3}$ or $f(x)=$ $(x+1) /(x-1)$ for $x \neq 1$.

A2: F-LE.A. 2
Given a graph, a description of a relationship, or two input-output pairs (include reading these from a table), construct linear and exponential functions, including arithmetic and geometric sequences to solve multistep problems.

## A2: F-LE.A. 4

For exponential models, express as a
logarithm the solution to $a b^{c t}=d$ where $a, c$, and $d$ are numbers and the base $b$ is 2,10 , or $e$; evaluate the logarithm using technology.

A1: F-LE.A. 1
Distinguish between situations that can be modeled with linear functions and with exponential functions.
a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

## A1: F-LE.A. 2

Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

## A2: A-SSE.B. 3

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
c. Use the properties of exponents to
transform expressions for exponential functions. For example, the expression $1.15^{t}$ can be rewritten as $\left(1.15^{1 / 12}\right)^{12 t} \approx$ $1.012^{12 t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is $15 \%$.

## A2: F-IF.C. 8

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
b. Use the properties of exponents to
interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y=$ $(1.02)^{\mathrm{t}}, \mathrm{y}=(0.97)^{\mathrm{t}}, \mathrm{y}=(1.01) 12^{\mathrm{t}}, \mathrm{y}=$
$(1.2)^{t} / 10$, and classify them as representing exponential growth or decay.

## A2: F-BF.A. 1

Write a function that describes a relationship between two quantities.
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

## A2: F-BF.A. 2

Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.


## A2: F-TF.B. 5

Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

## A2: F-TF.C. 8

Prove the Pythagorean identity $\sin ^{2}(\theta)+$ $\cos ^{2}(\theta)=1$ and use it to find $\sin (\theta), \cos (\theta)$, or $\tan (\theta)$ given $\sin (\theta), \cos (\theta)$, or $\tan (\theta)$ and the quadrant of the angle.

## A2: S-ID.A. 4

Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve

Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

## A2: F-TF.A. 2

Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

## 6.SP.B. 5

Summarize numerical data sets in relation to their context, such as by:
a. Reporting the number of observations.
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

## A1: S-ID.A. 2

Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

## A2: S-ID.B. 6

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize exponential models.

## A2: S-IC.A. 1

Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

## A1: S-ID.B. 6

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and quadratic models.
b. Informally assess the fit of a function by plotting and analyzing residuals.
c. Fit a linear function for a scatter plot that suggests a linear association.

## 7.SP.A. 2

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

## A2: F-BF.A. 1

Write a function that describes a relationship between two quantities.
a. Determine an explicit expression, a
recursive process, or steps for
calculation from a context.
b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

## A2: F-LE.A. 2

Given a graph, a description of a relationship, or two input-output pairs (include reading these from a table), construct linear and exponential functions, including arithmetic and geometric sequences to solve multistep problems.

A2: S-IC.A. 2
Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5 . Would a result of 5 tails in a row cause you to question the model?

## A2: S-IC.B. 3

Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how

## randomization relates to each.

## A2: S-IC.B. 4

Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
7.SP.C. 7

Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

## A2: S-IC.A. 2

Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5 . Would a result of 5 tails in a row cause you to question the model? A2: S-IC.B. 3
Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how
randomization relates to each.

A2: S-IC.B. 5
Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

## A2: S-IC.B. 6

Evaluate reports based on data

## A2: S-IC.A. 2

Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5 . Would a result of 5 tails in a row cause you to question the model?

## A2: S-IC.B. 3

Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

## A2: S-IC.B. 4

Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

## A2: S-IC.B. 5

Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

