High School Geometry

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LDOE TEACHER LEADER SUMMIT

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ERNEST MORIAL CONVENTION CENTER, NEW ORLEANS, LA

Agenda:

Fluency and Conceptual Understanding

Developing the Sequence of Instruction

Constructions-Transformations- Congruence

- Progressions from Grade 7 – Grade 8 – HS
- Conceptual Development through Hands-on and Technology
- Formative Assessment Task and On-line tools

Similarity – Circles – Trigonometry

- Progressions from Grade 7 – Grade 8 – HS
- Similarity of Circles
- Trigonometry

Introductions

Lori Fanning

Teach Geometry at LSU Lab School University High School

Have taught in Rapides Parish, Caddo Parish, and Fulton County Georgia

You are…..

Building Mathematical Fluency from Conceptual Understanding

"Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students become skillful in using procedures flexible as they solve contextual and mathematical problems."

Principles to Actions: Ensuring Mathematical Success for All

National Council Teachers of Mathematics 2014

The How

Teaching Deeper and Making Connections

- Procedural Fluency
- Conceptual Understanding
- Strategic Competence
- Reasoning
- Adaptations
- Productive Dispositions

Students with Conceptual Understanding…

- Know more than isolated facts and methods
- Know why a mathematical idea is important and the kinds of contexts in which it is useful
- Are able to learn new ideas by connecting them to ideas they already know
- Are able to remember or retain ideas….

Adding It Up, pg. 118

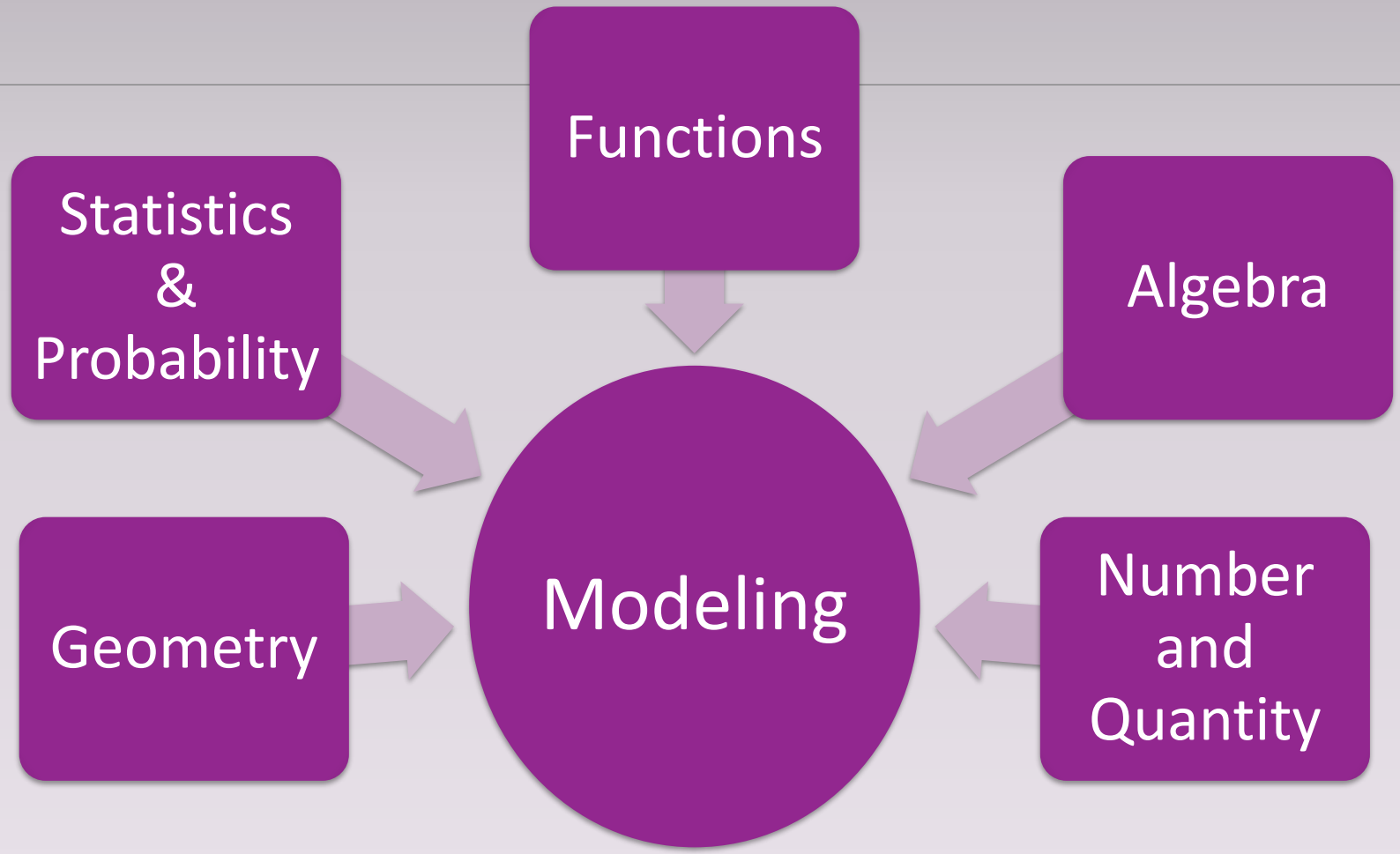
How is Conceptual Understanding Taught and Learned?

- Emphasis on BOTH IDEAS and skills
- Problem-Based Interactive Learning
- Emphasis on Connections

Developing Ideas

- Formatting – Organize it!!
- Multiple Representations – Draw it!!
- Connecting meaning to doing – Do it!!!

High School Conceptual Categories



The What

Developing the sequence of instruction

Unit 1 Introduction to Constructions and Elements of Geometry

Unit 2 Rigid Transformations and Congruence

Unit 3 Bisectors and Special Points in Triangle and Quadrilaterals

Unit 4 Dilations and Circles

Unit 5 Similarity

Unit 6 Probability

Unit 7 Trigonometry

Unit 8 Area and Surface Area

Unit 9 Volume and its applications

Unit 10 Analytical Geometry

The When

Louisiana Student Standards

GM: G-CO A Experiment with transformations in the plane

1. Precise definitions
2. Represent transformations in the plane
3. Describe rotations and reflections that carry figures on to themselves
4. Develop definition of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments
5. Given a figure draw a transformed figure using a sequence of transformations

Louisiana Student Standards

GM: G-CO B. Understand congruence in terms of rigid motions

5. Use the definition of congruence in terms of rigid motions to decide whether or not two figures are congruent.
6. Use the definition of congruence in terms of rigid motions to show that triangles are congruent.
7. Explain criteria for triangle congruence (ASA, SAS, SSS) that follow from the definition of congruence.

Geometry Lessons

Each lesson should

- Teach new knowledge
- Bring misconceptions to the surface
- Build skill or fluency
- Engage students in mathematical practice
- Instructional Tasks within lessons should build a balance of conceptual understanding and procedural skill
- Connect old learning to new learning

Constructions – Rigid Transformation - Congruence

Basic Constructions: patty paper, compass/straight edge
Copy and bisect an angle and Perpendicular bisector
Points of concurrency
Unknown angles: in planes, with transversals, in triangles
Exploring transformations with Geogebra
Proofs with constructions
Unknown angle proofs
Transformations and Rotations
Translations and the Perpendicular Bisector
Constructing Parallel Lines and Applying a Sequence of Rigid Motions
Proving Triangles Congruent

Geometry Progressions

Rigid Transformations and Congruence

THE MATH MARATHON

Preparing for the Gallery Walk

1. With your partner, take one of the problems from the Math Marathon off the wall.
2. Analyze the problem/task to determine to which grade level (7, 8, or G) tit belongs. Write the standard at the top of the chart.
3. Use markers to display the solution/work on the top half of the chart paper.
4. With your partner respond to the following questions/statements on the bottom half of your chart paper.
    a. What would be your mathematical goal for students as they complete this problem/task?
    b. What are the representations that you want to come out in a discussion to support the mathematics of this particular problem?
    c. Identify several other ways to support a productive struggle with scaffolding or manipulatives and chart them out as well?

Gallery Walk

With your partner, provide feedback (one ”grow ” and one “glow”) on sticky notes to each piece of work.

- Is the mathematics correct?
- Is the thinking visible?
- Is the solution justified?
- Would you answer the processing questions in the same way or can you add to the thinking of the participants?

Break (10 minutes)

Why take a transformational view of congruence and similarity?

- Precise vocabulary and language to describe how figures are alike or different
- Operationalizes the relationship between figures
- Allows an algebraic connection between figures under consideration
    - Similarity and slope and linearity
    - Area of similar figures
- Invokes and continues the idea of a functional view of all points in the plane

Why do we study transformations as part of congruence and similarity?

1. The foundational concept in modern geometry
2. Common in everyday experience.
3. Are fundamental in other parts of high-school math—not just geometry:
    a. Transformations are functions
    b. In the presence of coordinates, transformations can be expressed algebraically (by means of matrices).
    c. In algebra, one studies graphs by means of transformations,
    d. The notion of an abstract group is the most important idea in modern algebra
4. In geometry, transformations:
    a. Simplify the definitions of congruence and similarity
    b. Provide the language of symmetry

Congruence

Two figures are congruent if one can be mapped onto another by means of a rigid motion (translation, rotation, and/or reflection)

Vs.

Two figures are congruent if corresponding angles and sides are congruent

Constructions – Paper and Compass

Work in Pairs to complete the investigation on the hand-out.

Middle School Constructions

1. Duplicate a segment
2. Line through a given point parallel to a given line
3. Duplicate an angle
4. Bisect an angle
5. Perpendicular from a point to a line
6. Bisect a segment
7. Perpendicular through a point on a line
8. Equilateral triangle

High School Constructions

Get into groups of three: each person should choose a different method of construction: use patty paper; use a compass and straight edge; use geogebra on the computer/device. Construct the eight items below using appropriate labels, leaving construction marks. Glue your constructions into your interactive notebook or create an electronic interactive notebook.

1. Equilateral triangle	**5. Three angle bisectors of a triangle**
2. Construct an angle that is 4 times as large or ¼ as large as a given angle	6. Regular hexagon with side length 2 cm
3. Three perpendicular bisectors of the sides of a triangle	7. Square with 2cm diagonal.
4. Three altitudes of a triangle	8. Rhombus that is not a square with diagonal length of 2 cm.

Constructions With Exploration in Mind

Using Geogebra – See hand out

Formative Assessment

Paper pencil tasks Vs. Online tools

Instructional Task:
My Reflection

The Benefit of Open Ended Investigations

Draw a figure on the coordinate plane that can reflect on itself. Reflect it in the line y=x. Identify and label all of the congruent figures in your drawing.

Compare your figure with your neighbor. Use patty paper to show your neighbor that the figure reflects on itself.

What is the equation of the figures line of symmetry?

What do you notice about the line of symmetry and points that map on to each other?

Label a point on the reflection line outside of your figure point O. What is the length of the segment from point O to any two points that map onto each other?

On-line Assessment Tools

Quizziz.com

Plickers

Other Apps

Questions to ponder

How can I scaffold these tasks for students who are missing pre-requisite skills?

How can I extend these tasks for students who need an extension?

Summing it all up

- Transformations that translate, rotate, or reflect geometric figures preserve angle measure and length.
- Rigid transformations can be shown in the coordinate plane.
- Triangles are congruent if and only if there is a sequence of rigid motions that maps one triangle onto the other.

Lunch

Similarity

Two figures are similar if one is congruent to a dilation of another

Vs.

Two figures are similar if corresponding angles are congruent and sides are in proportion.

Louisiana Student Standards
Similarity, Right Triangles, and Trigonometry
G-SRT

A. Understand similarity in terms of similarity transformations.

1. Verify experimentally the properties of dilations given by a center and a scale factor:

a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Louisiana Student Standards
Similarity, Right Triangles, and Trigonometry
G-SRT

A. Understand similarity in terms of similarity transformations.

2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Louisiana Student Standards
Similarity, Right Triangles, and Trigonometry
G-SRT

B. Prove and apply theorems involving similarity.

4. Prove and apply theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity; SAS similarity criteria; SSS similarity criteria; ASA similarity.

5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Louisiana Student Standards, Right Triangles, and Trigonometry
G-SRT

C. Define trigonometric ratios and solve problems involving right triangles.

6. Understand that by similarity, side ratios in right triangles, including special right triangles (30-60-90 and 45-45- 90), are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

Dilations, Circles, and Trigonometry

Similarity Unit (includes proof of Pythagorean Theorem and special triangles)
Circles Unit: Dilations and Similarity of Circles
Equations and graphs of circles
Angles, Tangents, and Chords of circles
Segments of a circle
Arc Length and Sector Area
Trigonometry Unit: Trigonometric Ratios
Trigonometric Applications
Solving Right Triangles
Law of Sines and Cosines
Vectors

Geometry Progressions: Ratio/Proportions and Similarity

Ordering Task and Discussion

Louisiana Student Standards
Circles GM: G-C

A. Understand and apply theorems about circles.

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords, including the following: the relationship that exists between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; and a radius of a circle is perpendicular to the tangent where the radius intersects the circle.
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Louisiana Student Standards
Circles GM: G-C

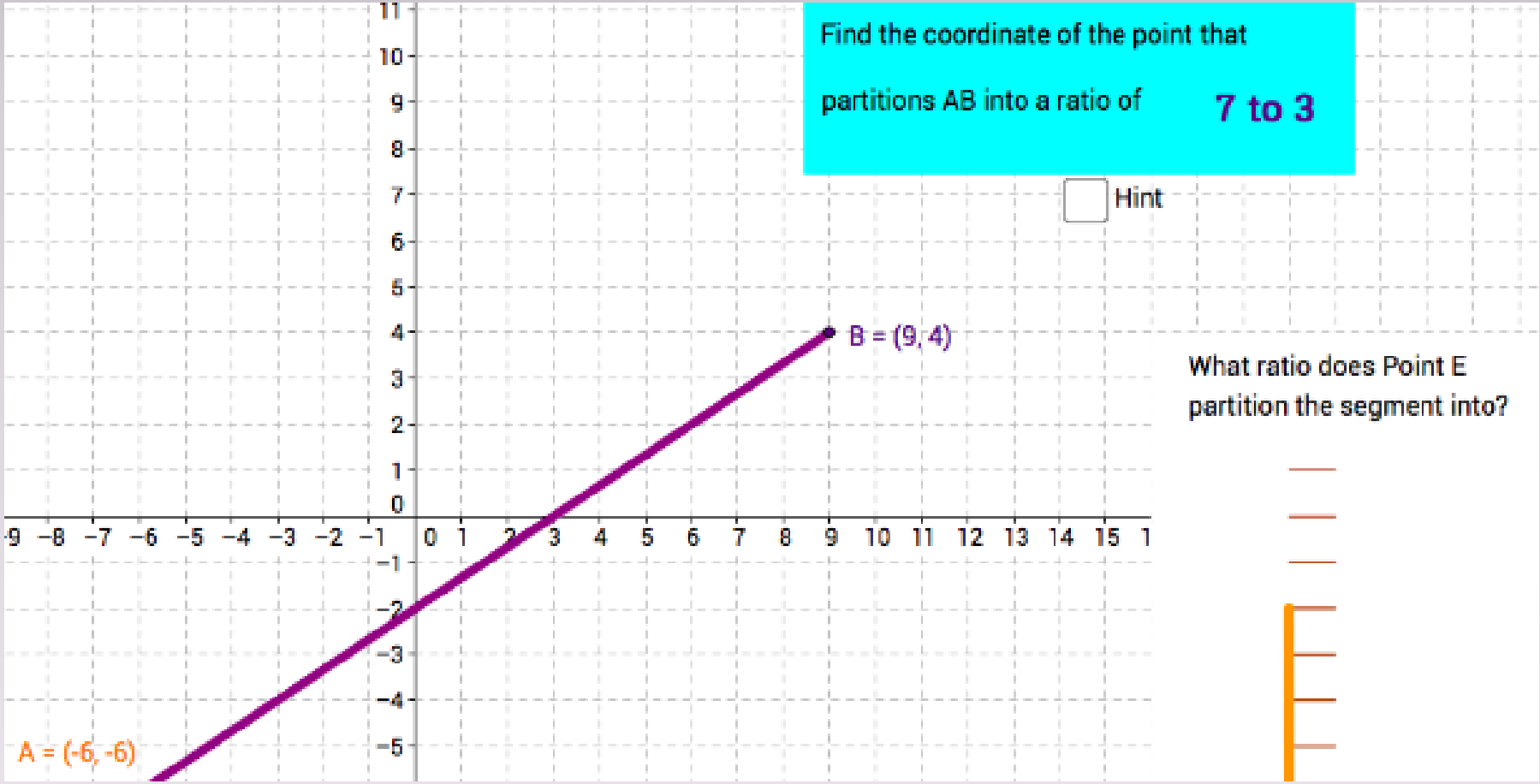
B. Find arc lengths and areas of sectors of circles.

4. Use similarity to determine that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Constructions with Exploration in Mind

See hand out

Instructional Task:
Partition a Segment See handout



Discussion Partitioning a Line Segment

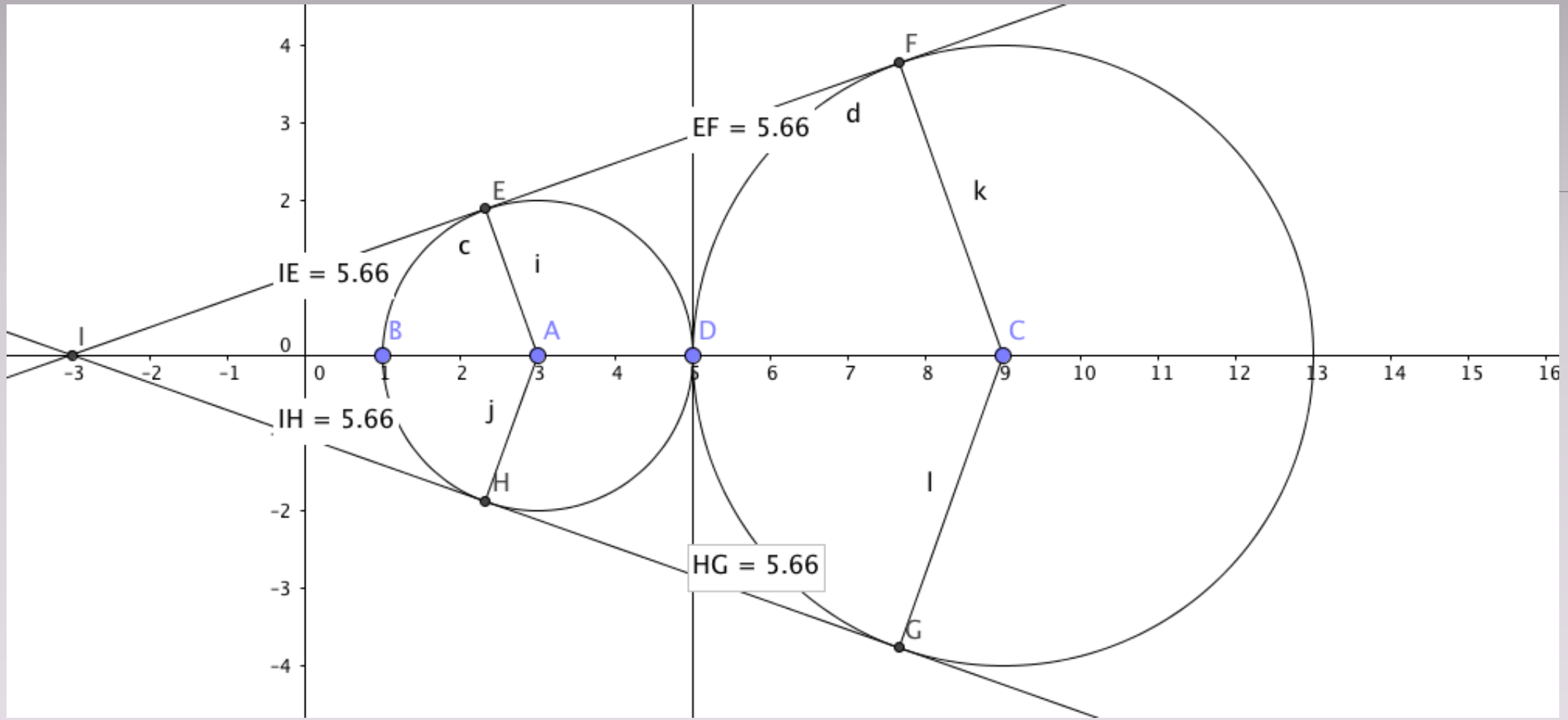
a. What is the slope of the line that contains segment AB?
b. What is the equation of the line that contains segment AB?
c. What is the scale factor used to partition the segment?
d. Name the point.

https://www.geogebra.org

Let’s Take a Break!!

Instructional Task:
Dilation of Circles

1. Graph Circle A with center at (3, 0) and radius 2. Dilate Circle A by applying a scale factor of 2 and and center of dilation P (-3, 0).
2. What is the ratio between the radii of the circles?
3. What is the ratio between the circumferences of the circles?
4. What is the ratio between the areas of the circles?
5. What do area ratios and circumference ratios have to do with scale factor?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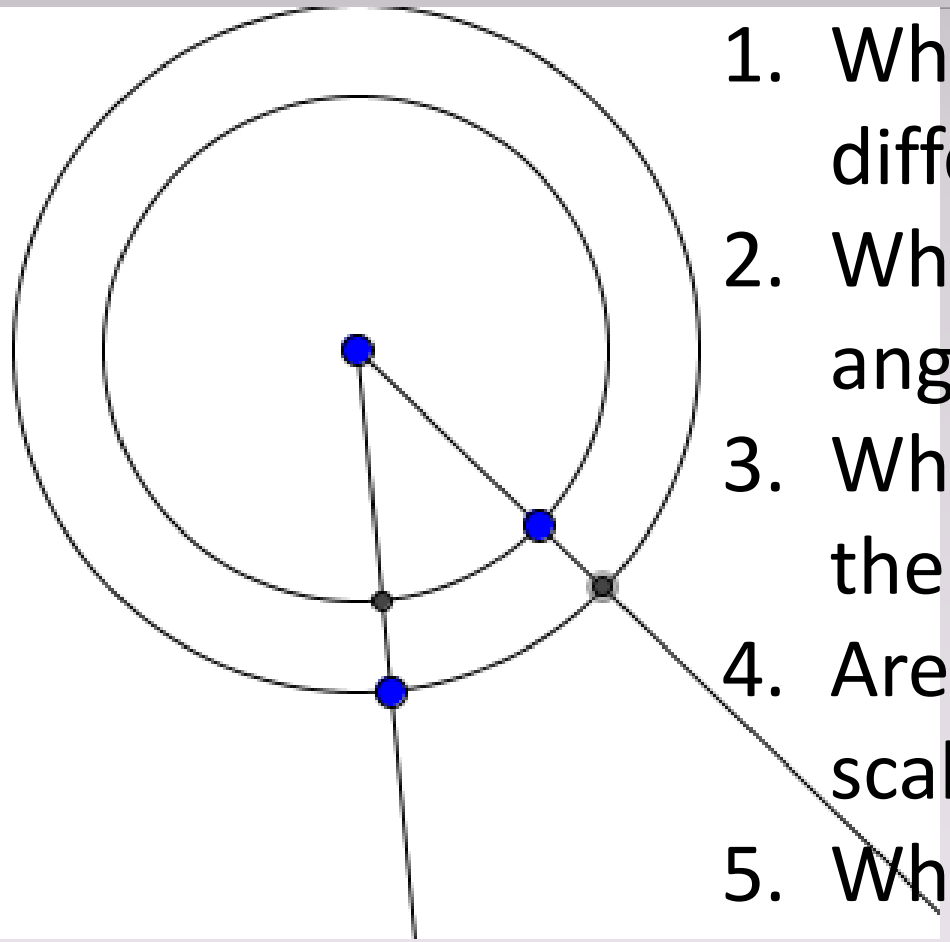
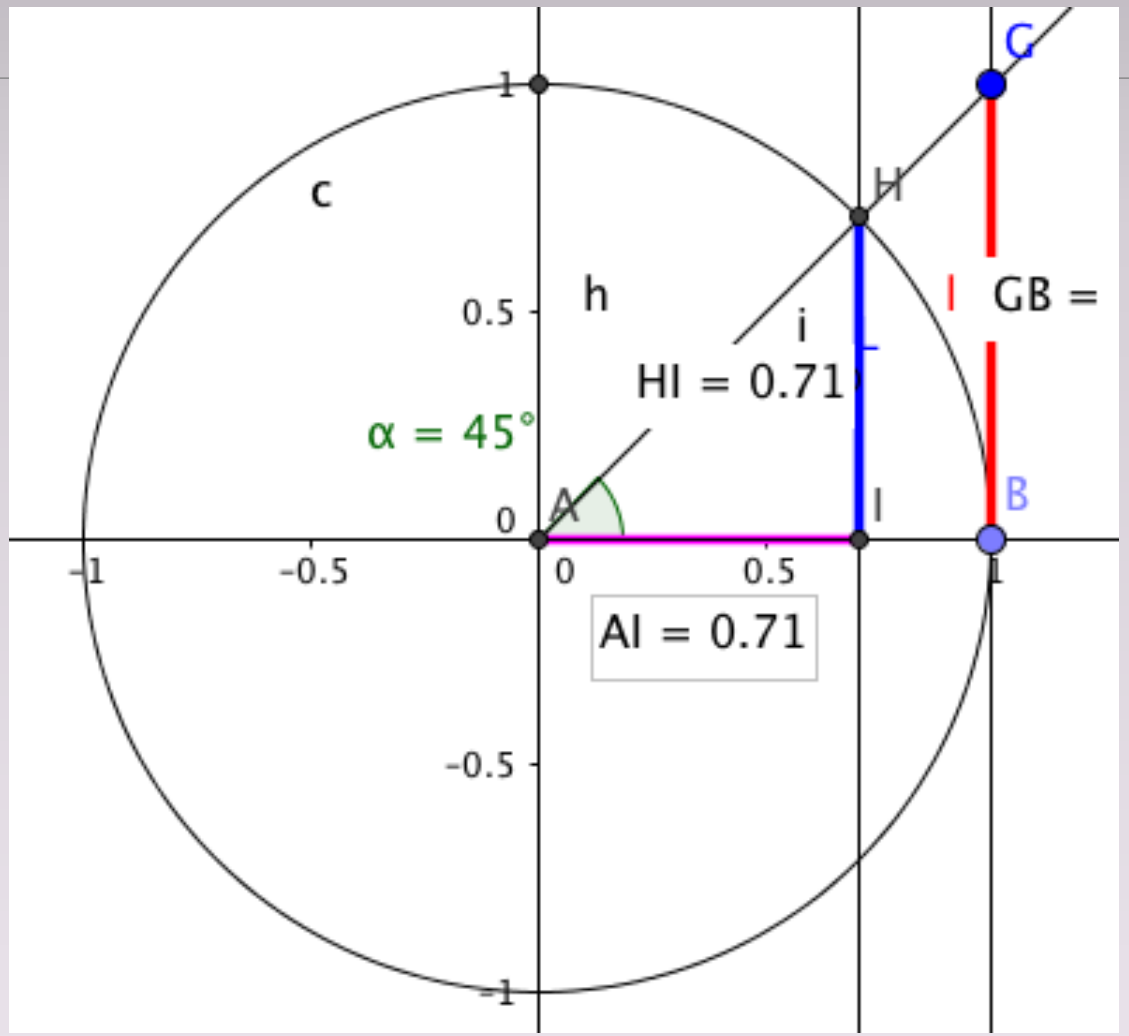
Instructional Task:
But What about the angles?

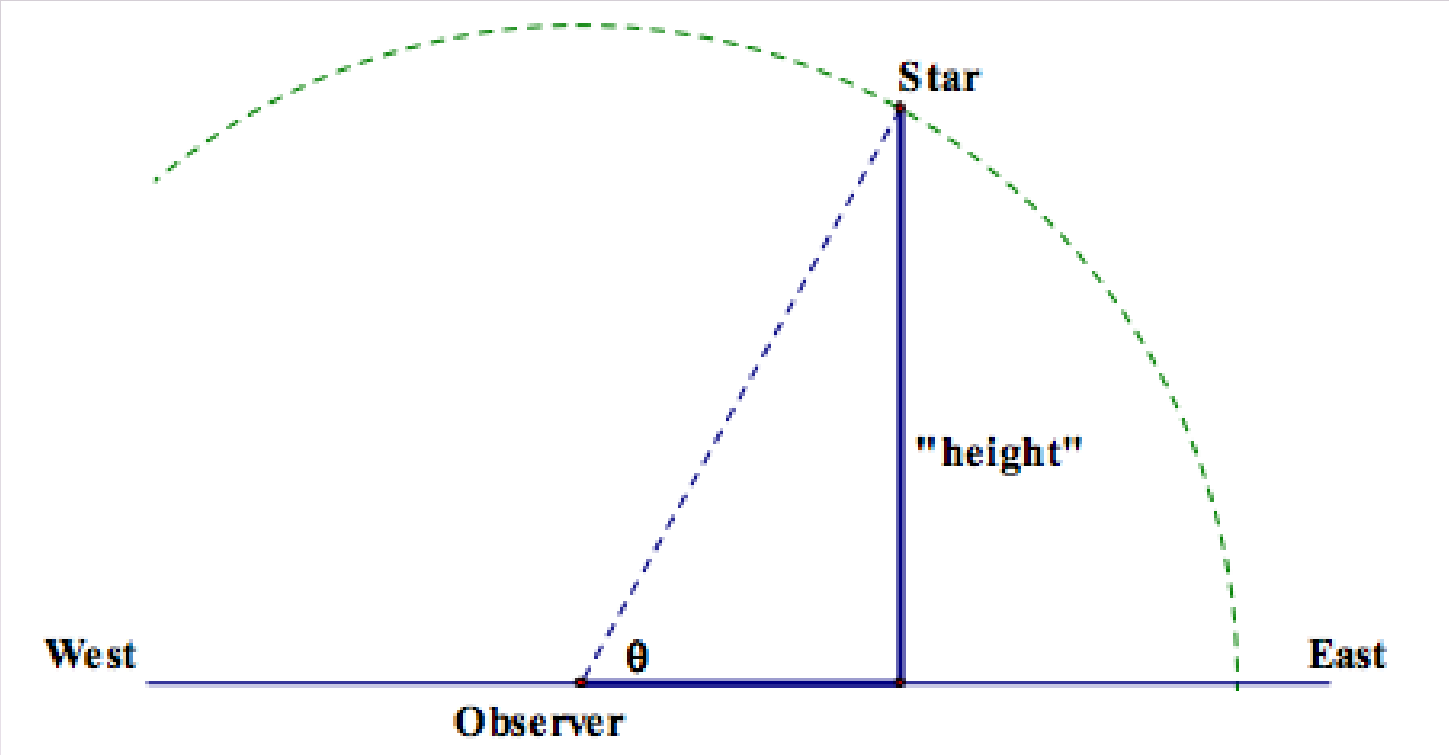


1. What is the ratio of the radii (you choose different radii for each circle?
2. What is the relationship between the central angles of the circle?
3. What is the ratio of the arc lengths cut off by the rays of the angles?
4. Are the circles Similar? If so, what is the scale factor and the center of dilation?
5. What is the ratio of the area of the sectors?

Tangents of Circles



A Little History



Trigonometry Tasks

See Hand out

Summary: Journal Entry

Name four instructional strategies we used today and describe how you can use them in your classroom?

What is your take-away?

How can you modify/extend what you learned today to improve your teaching in Geometry?

Sources

Mathematics Vision Project: Transforming Mathematics Education

Illustrative Mathmatics (https://www.illustrativemathematics.org)

Eureka Math (http://greatminds.net/maps/math/home)

Louisiana Eagle (https://www.louisianaeagle.org/griffin/#login)

Louisiana DOE Mathematics Guidebook (http://www.louisianabelieves.com/docs/default-source/teacher-toolbox-resources/2014)

Mathematics Assessment Project (http://map.mathshell.org)