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Algebra II

**Louisiana Student Standards: Companion Document for Teachers**

This document is designed to assist educators in interpreting and implementing Louisiana’s new mathematics standards. It contains descriptions of each Algebra II standard to answer questions about the standard’s meaning and how it applies to student knowledge and performance. To ensure that descriptions are helpful and meaningful to teachers, this document also provides examples indicating how students might meet the requirements of a standard. Examples are samples only and should not be considered an exhaustive list.

This companion document is considered a “living” document as we believe that teachers and other educators will find ways to improve the document as they use it. Please send feedback to LouisianaStandards@la.gov so that we may use your input when updating this guide.

Additional information on the Louisiana Student Standards for Mathematics, including how to read the standards’ codes, a listing of standards for each grade or course, and links to additional resources, is available at

<http://www.louisianabelieves.com/resources/library/k-12-math-year-long-planning>.

Updated September 21, 2016. Click [here](#Updates) to view updates.



**Standards for Mathematical Practices**

The Louisiana Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students in grades K-12. Below are a few examples of how these practices may be integrated into tasks that high school students complete.

| Louisiana Standards for Mathematical Practice (MP) for High School |
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| **Louisiana Standard** | **Explanations and Examples** |
| **HS.MP.1.** Make sense of problems and persevere in solving them. | High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. |
| **HS.MP.2.** Reason abstractly and quantitatively. | High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects. |
| **HS.MP.3.** Construct viable arguments and critique the reasoning of others. | High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains, to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. |
| **HS.MP.4. Model with mathematics.** | High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. |
| **HS.MP.5.** Use appropriate tools strategically. | High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts. |
| **HS.MP.6.** Attend to precision. | High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and label axes to clarify the correspondence between quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions. |
| **HS.MP.7.** Look for and make use of structure. | By high school, students look closely to discern a pattern or structure. In the expression *x*2 + 9*x* + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(*x* – *y*)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures. |
| **HS.MP.8.** Look for and express regularity in repeated reasoning. | High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding (*x* – 1)(*x* + 1), (*x* – 1)(*x*2 + *x* + 1), and (*x* – 1)(*x*3 + *x*2 + *x* + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. |

**Modeling Standards**

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

**What is Modeling**

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

* Estimate how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
* Plan a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
* Design the layout of the stalls in a school fair so as to raise as much money as possible.
* Analyze the stopping distance for a car.
* Model a savings account balance, bacterial colony growth, or investment growth.
* Engage in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
* Analyze the risk in situations such as extreme sports, pandemics, and terrorism.
* Relate population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters which are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g**.,** the behavior of polynomials) as well as physical phenomena)**.**

| Number and Quantity: The Real Number System (N-RN)**Extend the properties of exponents to rational exponents.** |
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| **Louisiana Standard** | **Explanations and Examples** |
| **A2: N-RN.A.1.** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.* | The meaning of an exponent relates the frequency with which a number is used as a factor. So indicates the product where 5 is a factor 3 times. Extend this meaning to a rational exponent, then indicates one of three equal factors whose product is 125.Students recognize that a fractional exponent can be expressed as a radical or a root. *For example*, an exponent of a is equivalent to a cube root; an exponent of is equivalent to a fourth root. Students extend the use of the power rule, from whole number exponents i.e., to rational exponents. They compare examples, such as to to establish a connection between radicals and rational exponents: and, in general, = . **Examples:*** Determine the value of *x*
* A biology student was studying bacterial growth. The population of bacteria doubled every hour as indicated in the following table:

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| --- | --- | --- | --- | --- | --- |
| # of hours of observation | 0 | 1 | 2 | 3 | 4 |
| Number of bacteria cells (thousands) | 4 | 8 | 16 | 32 | 64 |

How could the student predict the number of bacteria every half hour? every 20 minutes?*Solution:* If every hour the number of bacteria cells is being multiplied by a factor of 2 then on the half hour the number of cells is increasing by a factor of . For every 20 minutes, the number of cells is increasing by a factor of . |
| **A2: N-RN.A.2.** Rewrite expressions involving radicals and rational exponents using the properties of exponents. | Students rewrite expressions involving rational exponents as expressions involving radicals and simplify those expressions.**Examples:*** **Using** the properties of exponents, simplify
* What is an equivalent exponential expression for ? Explain how they are equivalent.

*Solution****:*** In the first expression, the base number is 8 and the exponent is . This means that the expression represents 2 of the 3 equal factors whose product is 8, thus the value is 4, since ; there are three factors of 2; and two of these factors multiply to be 4. In the second expression, there are 2 equal factors of 8 or 64. The exponent represents 1 of the 3 equal factors of 64. Since then one of the three factors is 4. The last expression there is 1 of 3 equal factors of 8 which is 2 since. Then there are 2 of the equal factors of 2, which is 4.Students rewrite expressions involving radicals as expressions using rational exponents and use the properties of exponents to simplify the expressions.**Example:*** Given , which form would be easiest to calculate without using a calculator. Why?
* Determine whether each equation is true or false using the properties of exponents. If false, describe at least one way to make the math statement true. Justify

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| Number and Quantity: The Real Number System (N-RN)**Use properties of rational and irrational numbers.** |
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| **Louisiana Standard** | **Explanations and Examples** |
| **A2: N-Q.A.2.** Define appropriate quantities for the purpose of descriptive modeling. | **Examples:*** What type of measurements would one use to determine their income and expenses for one month?
* How could one express the number of accidents in Louisiana?
* Explain how the units cm, cm2, and cm3 are related and how they are different. Describe situations where each would be an appropriate unit of measure.
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| Number and Quantity: The Complex Number System (N-CN)**Perform arithmetic operations with complex numbers.** |
| **Louisiana Standard** | **Explanations and Examples** |
| **A2: N-CN.A.1.** Know there is a complex number *i* such that *i*2 = −1, and every complex number has the form *a* + *bi* with *a* and *b* real. | Students will review the structure of the complex number system realizing that every number is a complex number that can be written in the form where *a* and *b* are real numbers. If , then the number is a pure imaginary number however when the number is a real number. The square root of a negative number is a complex number. **Example**:

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|  | **Problem** | **Solution** | ***bi* Form** |
| 1. |  |  |  |
| 2. |  |  |  |
| 3. |  |  |  |
| 4. |  |  |  |

**Example:** Explore the powers of and apply a pattern to simplfy  |
| **A2: N-CN.A.2.** Use the relation *i*2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. | The complex number system possesses the same basic properties as the real number system: that addition and multiplication are commutative and associative; the existence of additive identity and multiplicative identity; the existence of an additive inverse for every complex number and the existence of multiplicative inverse or reciprocal for every non- zero complex number; and the distributive property of multiplication over the addition. An awareness of the properties minimizes students’ rote memorization and links the rules for manipulations with the complex number system to the rules for manipulations with binomials with real coefficients of the form .**Examples:*** Simplify the following expression. Justify each step using the commutative, associative and distributive properties.

Solutions may vary; one solution follows:* Ohms’ Law related the voltage E, current I, and resistence R, in an electrical circuit: . Respectively, these quantities are measured in volts, amperes, and ohms.
1. Find the voltage in an electrical circuit with current amperes and resistance ohms.
2. Find the necessary resistance value to produce a voltage that is not complex.
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| Number and Quantity: The Complex Number System (N-CN)**Use complex numbers in polynomial identities and equations.** |
| **Louisiana Standard** | **Explanations and Examples** |
| **A2: N-CN.C.7.** Solve quadratic equations with real coefficients that have complex solutions. | Students determine when a quadratic equation in standard form, , has complex roots by looking at a graph of or by calculating the discriminant.**Examples:*** Within which number system can *x*2 = – 2 be solved? Explain how you know.
* Solve *x*2+ 2*x* + 2 = 0 over the complex numbers.
* Find all solutions of 2*x*2 + 5 = 2*x* and express them in the form *a* + *bi.*
* Given the quadratic equation that has a solution of , determine possible values for *a*, *b*,and *c*. Are there other combinations possible? Explain.
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| Algebra: Seeing Structure in Expressions (A-SSE)**Interpret the structure of expressions.** |
| **Louisiana Standard** | **Explanations and Examples** |
| **A2: A-SSE.A.2.** Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).* | Students should extract the greatest common factor (whether a constant, a variable, or a combination of each). If the remaining expression is quadratic, students should factor the expression further.**Example:*** Factor
* Rewrite into an equivalent form.
* Factor
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| Algebra: Seeing Structure in Expressions (A-SSE)**Write expressions in equivalent forms to solve problems.** |
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| **Louisiana Standard** | **Explanations and Examples** |
| **A2: A-SSE.B.3.** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **★**c. Use the properties of exponents to transform expressions for exponential functions*. For example the expression 1.15t can be rewritten as (1.151/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* | Students will use the properties of operations to create equivalent expressions.**Example:**Forms of Exponential Expressions: <https://www.illustrativemathematics.org/content-standards/HSA/SSE/B/3/tasks/1305>  |
| **A2: A-SSE.B.4.** Apply the formula for the sum of a finite geometric series (when the common ratio is not 1) to solve problems. *For example, calculate mortgage payments.* | This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. Students understand that a geometric series is the sum of terms in a geometric sequence and can be used to solve real-world problems. The sum of a finite geometric series with common ratio not equal to 1 can be written as the simple formula where *r* is the common ratio, *a* is the initial value, and *n* is the number of terms in the series.**Examples:*** In February, the Bezanson family starts saving for a trip to Australia in September. The Bezanson’s expect their vacation to cost $5375. They start with $525. Each month they plan to deposit 20% more than the previous month. Will they have enough money for their trip?
* An amount of $100 was deposited in a savings account on January 1st each of the years 2010, 2011, 2012, and so on to 2019, with annual yield of 7%. What will be the balance in the savings account on January 1, 2020?
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| Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)**Perform arithmetic operations on polynomials.** |
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| **Louisiana Standard** | **Explanations and Examples** |
| **A2: A-APR.B.2.** Know and apply the Remainder Theorem: For a polynomial *p*(*x*) and a number *a*, the remainder on division by *x* – *a* is *p*(*a*), so *p*(*a*) = 0 if and only if (*x* – *a*) is a factor of *p*(*x*). | The Remainder theorem says that if a polynomial *p*(*x*) is divided by *x* – *a*, then the remainder is the constant *p*(*a*). That is, So if p(a) = 0 then p(x) = q(x)(x-a).**Example:*** Let. Evaluate p(-2). What does your answer tell you about the factors of p(x)? [Answer: p(-2) = 0 so x+2 is a factor.]
* Consider the polynomial function: , where *a* is an unknown real number. If is a factor of this polynomial, what is the value of *a*?
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| **A2: A-APR.B.3.** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. | Students identify the multiplicity of the zeroes of a factored polynomial and explain how the multiplicity of the zeroes provides a clue as to how the graph will behave when it approaches and leaves the x-intercept. Graphing calculators or programs can be used to generate graphs of polynomial functions.**Example:*** Factor the expression and explain how your answer can be used to solve the equation. Explain why the solutions to this equation are the same as the x-intercepts of the graph of the function .
* For a certain polynomial function, is a zero with multiplicity two, is a zero with multiplicity three, and is a zero with multiplicity one. Write a possible equation for this function and sketch its graph.
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| Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)**Use polynomial identities to solve problems.** |
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| **Louisiana Standard** | **Explanations and Examples** |
| **A2: A-APR.C.4.** Use polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* *(x2+y2)2 = (x2– y2)2 + (2xy)2 can be used to generate Pythagorean triples.* | Polynomial identities should include but are not limited to:* The product of the sum and difference of two terms,
* The difference of two squares,
* The sum and difference of two cubes,
* The square of a binomial

**Examples:*** Use the distributive law to explain why *x*2 – *y*2 = (*x* – *y*)(*x* + *y*) for any two numbers *x* and *y*.
* Derive the identity (*x* – *y*)2 = *x*2 – 2*xy* + *y*2from (*x* + *y*)2 = *x*2 + 2*xy* + *y*2 by replacing *y* by –*y.*
* Use an identity to explain the pattern

22 – 12 = 332 – 22 = 542 – 32 = 752 – 42 = 9[Answer: (*n* + 1)2 - *n*2 = 2*n* + 1 for any whole number *n*.] |

| Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)**Rewrite rational expressions.** |
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| **Louisiana Standard** | **Explanations and Examples** |
| **A2: A-APR.D.6.** Rewrite simple rational expressions in different forms; write *a*(*x*)/*b*(*x*) in the form *q*(*x*) + *r*(*x*)/*b*(*x*), where *a*(*x*), *b*(*x*), *q*(*x*), and *r*(*x*) are polynomials with the degree of *r*(*x*) less than the degree of *b*(*x*), using inspection, long division, or, for the more complicated examples, a computer algebra system. | Rewrite rational expressions, , in the form by using inspection (factoring) or long division. The polynomial *q*(*x*) is called the quotient and the polynomial *r*(*x*) is called the remainder. Expressing a rational expression in this form allows one to see different properties of the graph, such as horizontal asymptotes.**Examples:*** Find the quotient and remainder for the rational expression and use them to write the expression in a different form.

Students determine the best method of simplifying a given rational expression.***Example (using inspection):*** , ***Example (long division):*** , *Note: The use of synthetic division may be introduced as a method but students should recognize its limitations (division by a linear term). When students use methods that have not been developed conceptually, they often create misconceptions and make procedural mistakes due to a lack of understanding as to why the method is valid. They also lack the understanding to modify or adapt the method when faced with new and unfamiliar situations. Suggested viewing Synthetic Division: How to understand It by not doing it.* [*http://www.youtube.com/watch?v=V-Q6jBYn3Oc*](http://www.youtube.com/watch?v=V-Q6jBYn3Oc) |

| Algebra: Creating Equations ★ (A-CED)**Create equations that describe numbers or relationships.** |
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| **Louisiana Standard** | **Explanations and Examples** |
| **A2: A-CED.A.1.** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* | Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth.**Examples:*** Given that the following trapezoid has area 54 cm2, set up an equation to find the length of the base, and solve the equation.

trap.gif* Lava coming from the eruption of a volcano follows a parabolic path. The height *h* in feet of lava *t* seconds after it is ejected from the volcano is given by After how many seconds does the lava reach its maximum height of 1000 feet?
* Chase and his brother like to play basketball. About a month ago they decided to keep track of how many games they have each won. As of today, Chase has won 18 out of the 30 games against his brother.
* How many games would Chase have to win in a row in order to have a 75% winning record?
* How many games would Chase have to win in a row in order to have a 90% winning record?
* Is Chase able to reach a 100% winning record? Explain why or why not.
* Suppose that after reaching a winning record of 90% in part (b), Chase had a losing streak. How many games in a row would Chase have to lose in order to drop down to a winning record below 55% again?
* If the world population at the beginning of 2008 was 6.7 billion and growing at a rate of 1.16% each year, in what year will the population be double?
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| Algebra: Creating Equations ★ (A-REI)**Understand solving equations as a process of reasoning and explain the reasoning.** |
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| **Louisiana Standard** | **Explanations and Examples** |
| **A2: A-REI.A.1.** Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. | Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. In addition, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions. Other operations, such as squaring both sides, may produce equations that have extraneous solutions.**Examples:*** Show that *x* = 2 and *x* = -3 are solutions to the equation Write the equation in a form that shows these are the only solutions, explaining each step in your reasoning.
* Prove . Justify each step.
* Explain the steps involved in solving each of the following:

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| 1.
 | 1.
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| **A2: A-REI.A.2.** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. | **Examples:*** Mary solved for *x* and got and . Evaluate her solutions and determine if she is correct. Explain your reasoning.
* Solve . Can *x* have a value of 3? Explain your reasoning.
* Solve for x:
	+
	+
	+
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| Algebra: Creating Equations ★ (A-REI)**Solve equations and inequalities in one variable.** |
| **Louisiana Standard** | **Explanations and Examples** |
| **A2: A-REI.B.4.** Solve quadratic equations in one variable.1. Solve quadratic equations by inspection (e.g., for *x*2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as *a* ± *bi* for real numbers *a* and *b*.
 | Students should solve by factoring, completing the square, and using the quadratic formula. The zero product property is used to explain why the factors are set equal to zero. Students should relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to *ax*2 + *bx* + *c*  = 0 to the behavior of the graph of *y*  = *ax*2 + *bx* + *c*.

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| **Value of Discriminant** | **Nature of Roots** | **Nature of Graph** |
| *b*2 – 4*ac* = 0 | 1 real roots | intersects *x*-axis once |
| *b*2 – 4*ac* > 0  | 2 real roots | intersects *x*-axis twice |
| *b*2 – 4*ac* < 0 | 2 complex roots | does not intersect *x*-axis |

**Examples:*** Are the roots of 2x2 + 5 = 2x real or complex? How many roots does it have? Find all solutions of the equation.
* What is the nature of the roots of x2 + 6x + 10 = 0? Solve the equation using the quadratic formula and completing the square. How are the two methods related?
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| Algebra: Creating Equations ★ (A-REI)**Solve systems of equations.** |
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| **Louisiana Standard** | **Explanations and Examples** |
| **A2: A-REI.C.6.** Solve systems of linear equations exactly and approximately (e.g., with graphs), limited to systems of at most three equations and three variables. With graphic solutions, systems are limited to two variables. | The system solution methods can include but are not limited to graphical, elimination/linear combination, substitution, and modeling. Systems can be written algebraically or can be represented in context. Students may use graphing calculators, programs, or applets to model and find approximate solutions for systems of equations. **Examples:** * José had 4 times as many trading cards as Phillipe. After José gave away 50 cards to his little brother and Phillipe gave 5 cards to his friend for this birthday, they each had an equal amount of cards. Write a system to describe the situation and solve the system.

* Solve the system of equations: *x*+ *y* = 11 and 3*x* – *y* = 5.
* Use a second method to check your answer.
* Solve the system of equations: *x* – 2*y +* 3*z* = 5, *x* + 3*z* = 11, 5*y* – 6*z* = 9.
* The opera theater contains 1,200 seats, with three different prices. The seats cost $45 per seat, $50 per seat, and $60 per seat. The opera needs to gross $63,750 on seat sales. There are twice as many $60 seats as $45 seats. How many seats in each level need to be sold?
 |

| **A2: A-REI.C.7.** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y = –3x and the circle x2 + y2 = 3.* | **Example:*** Two friends are driving to the Grand Canyon in separate cars. Suzette has been there before and knows the way but Andrea does not. During the trip Andrea gets ahead of Suzette and pulls over to wait for her. Suzette is traveling at a constant rate of 65 miles per hour. Andrea sees Suzette drive past. To catch up, Andrea accelerates at a constant rate. The distance in miles (*d*) that her car travels as a function of time in hours (*t*) since Suzette’s car passed is given by *d* = 3500*t*2.

Write and solve a system of equations to determine how long it takes for Andrea to catch up with Suzette. |
| --- | --- |
| Algebra: Creating Equations ★ (A-REI)**Represent and solve equations and inequalities graphically.** |
| **Louisiana Standard** | **Explanations and Examples** |
| **A2: A-REI.D.11.** Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions.**Example:** * Graph the following system and approximate solutions for

 and * Let and . Determine solution(s) for . Explain what the solution(s) mean in terms of the functions given.
* Use technology to solve , treating each side of the statement as two separate equations.
 |

| Functions: Interpreting Functions (F-IF)**Interpret functions that arise in applications in terms of the context.** |
| --- |
| **Louisiana Standard** | **Explanations and Examples** |
| **A2: F-IF.B.4.** For linear, piecewise linear (to include absolute value), quadratic, and exponential functions that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior. **★** | Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology. **Examples:** * A rocket is launched from 180 feet above the ground at time *t* = 0. The function that models this situation is given by *h* = – 16*t*2 + 96*t* + 180, where *t* is measured in seconds and *h* is height above the ground measured in feet.
* What is a reasonable domain restriction for *t* in this context?
* Determine the height of the rocket two seconds after it was launched.
* Determine the maximum height obtained by the rocket.
* Determine the time when the rocket is 100 feet above the ground.
* Determine the time at which the rocket hits the ground.
* How would you refine your answer to the first question based on your response to the second and fifth questions?
* Compare the graphs of *y* = 3*x*2 and *y* = 3*x*3.
* Let . Find the domain of *R*(*x*). Also find the range, zeros, and asymptotes of *R*(*x*).
* Let . Graph the function and identify end behavior and any intervals of constancy, increase, and decrease.
* For the function on the right, label and describe the key features. Include intercepts, relative max/min, intervals of increase/decrease, and end behavior.
 |
| **A2: F-IF.B.4.** *continued* | * Number of customers at a coffee shop vary throughout the day. The coffee shop opens at 5:00am and number of customers increase slowly at first and increase more and more until reaching a maximum number of customers for the morning at 8:00 am. Number of customers slowly decrease until 9:30 when they drop significantly and then remain steady until 11:00 am when the lunch crowd begins to show. Similar to the morning, the number of customers increase slowly and then begin to increase more and more. The maximum customers is less at lunch than breakfast and is largest at 12:20pm. The smallest number of customers since opening occurs at 2:00 pm. There is a third spike in customers around 5:00 pm and then a late night crowd around 9:00 pm before closing at 10:00 pm. Sketch a graph that would model the number of customers at the coffee shop during the day.
* Over a year, the length of the day (the number of hours from sunrise to sunset) changes every day. The table below shows the length of day every 30 days from 12/31/97 to 3/26/99 for Boston Massachusetts.

During what part of the year do the days get longer? Support your claim using information provided from the table.**Sample Graph*** Jumper horses on carousels move up and down as the carousel spins. Suppose that the back hooves of such a horse are six inches above the floor at their lowest point and two-and-one-half feet above the floor at their highest point. Draw a graph that could represent the height of the back hooves of this carousel horse during a half-minute portion of a carousel ride.
 |

| **A2: F-IF.B.6.** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **★** | The average rate of change of a function *y* = *f*(*x*) over an interval [a,b] isIn addition to finding average rates of change from functions given symbolically, graphically, or in a table, Students may collect data from experiments or simulations (ex. falling ball, velocity of a car, etc.) and find average rates of change for the function modeling the situation.**Examples:*** Use the following table to find the average rate of change of *g* over the intervals [-2, -1] and [0,2]:

|  |  |
| --- | --- |
| ***x*** | ***g(x)*** |
| -2 | 2 |
| -1 | -1 |
| 0 | -4 |
| 2 | -10 |

* The plug is pulled in a small hot tub. The table gives the volume of water in the tub from the moment the plug is pulled, until it is empty. What is the average rate of change between:
* 60 seconds and 100 seconds?
* 0 seconds and 120 seconds?
* 70 seconds and 110 seconds?

  |

| Functions: Interpreting Functions (F-IF)**Analyze functions using different representations.** |
| --- |
| **Louisiana Standard** | **Explanations and Examples** |
| **A2: F-IF.C.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **★**1. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
2. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
3. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
 | Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions.**Examples:*** Describe key characteristics of the graph of *f(x) =* │*x –* 3│ +5.
* Sketch the graph and identify the key characteristics of the function described below.

 * Graph the function *f*(*x*) *= 2x* by creating a table of values. Identify the key characteristics of the graph.
 |

| **A2: F-IF.C.8.** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.1. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*
 | Students can determine if an exponential function models growth or decay. Students can also identify and interpret the growth or decay factor. Students can rewrite an expression in the form as . They can identify as the growth or decay factor.Students recognize that when the factor is greater than 1, the function models growth and when the factor is between 0 and 1 the function models decay.**Examples:*** The projected population of Delroysville is given by the function where *t* is the number of years since 2010. You have been selected by the city council to help them plan for future growth. Explain what the function means to the city council members.
* Suppose a single bacterium lands on one of your teeth and starts reproducing by a factor of 2 every hour. If nothing is done to stop the growth of the bacteria, write a function for the number of bacteria as a function of the number of days.
* The expression represents the amount of a drug in milligrams that remains in the bloodstream after *x* hours.
1. Describe how the amount of drug in milligrams changes over time.
2. What would the expression represent?
3. What new or different information is revealed by the changed expression?
 |
| --- | --- |
| **A2: F-IF.C.9.** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | Students compare properties of two functions. The representations of the functions should vary: table, graph, algebraically, or verbal description.**Example:*** If and is represented on the graph.

What is the difference between the zero with the least value of and the zero with the least value of Which has the largest relative maximum?Describe their end behaviors. Why are they different? What can be said about each function? |

| Functions: Building Functions (F-BF)**Build a function that models a relationship between two quantities.** |
| --- |
| **Louisiana Standard** | **Explanations and Examples** |
| **A2: F-BF.A.1.** Write a function that describes a relationship between two quantities.**★**1. Determine an explicit expression, a recursive process, or steps for calculation from a context.
2. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
 | Students will analyze a given problem to determine the function expressed by identifying patterns in the function’s rate of change. They will specify intervals of increase, decrease, constancy, and, if possible, relate them to the function’s description in words or graphically. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions.**Examples:*** You buy a $10,000 car with an annual interest rate of 6 percent compounded annually and make monthly payments of $250. Express the amount remaining to be paid off as a function of the number of months, using a recursion equation.
* The radius of a circular oil slick after *t* hours is given in feet by , for 0 ≤ *t* ≤ 10. Find the area of the oil slick as a function of time.
* A cup of coffee is initially at a temperature of 93º F. The difference between its temperature and the room temperature of 68º F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time.
* Suppose you deposit $100 in a savings account that pays 4% interest, compounded annually. At the end of each year you deposit an additional $50. Write a recursive function that models the amount of money in the account for any year.
 |
| **A2: F-BF.A.2.** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.**★** | An explicit rule for the *n*th term of a sequence gives *a*n as an expression in the term’s position *n*; a recursive rule gives the first term of a sequence, and a recursive equation relates *a*n to the preceding term(s). Both methods of presenting a sequence describe *a*n as a function of *n*.**Examples:*** Generate the 5th-11th terms of a sequence if *A*1= 2 and
* Use the formula: *An*= *A*1 + *d*(*n* - 1) where d is the common difference to generate a sequence whose first three terms are: -7, -4, and -1.
* Given the formula *An*= 2*n* - 1, find the 17th term of the sequence. What is the 9th term in the sequence 3, 5, 7, 9, …?
* Given *a1*= 4 and *an*= *an-1*+ 3, write the explicit formula.
* Given the sequence defined by the function with . Write an explicit function rule.
 |

| **A2: F-BF.B.3.** Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* | Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions.**Examples:*** Is *f*(*x*) = x3 - 3x2 + 2x + 1 even, odd, or neither? Explain your answer orally or in written format.

**parabola graph*** Describe effect of varying the parameters *a, h,* and *k* have on the shape and position of the graph of f(x) = a(x-h)2 + k.
* Compare the shape and position of the graphs of  to , and explain the differences, orally or in written format, in terms of the algebraic expressions for the functions.

transform e |
| --- | --- |
| **A2: F-BF.B.3.** *continued* | * Compare the shape and position of the graphs of *y* = sin to *y* = 2 sin
 |
| **A2: F-BF.B.4** Find inverse functions.1. Solve an equation of the form *f*(*x*) = *c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x-1) for x ≠ 1.*
 | Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions.**Examples:*** For the function *h*(*x*) = (*x* – 2)3, defined on the domain of all real numbers, find the inverse function if it exists or explain why it doesn’t exist.
* Find the inverse of the function and demonstrate it is the inverse using input – output pairs.
* Find a domain for f(*x*) = 3*x*2 + 12*x* - 8 on which it has an inverse. Explain why it is necessary to restrict the domain of the function.
 |

| Functions: Linear, Quadratic, and Exponential Models (F-LE)**Construct and compare linear, quadratic, and exponential models and solve problems.** |
| --- |
| **Louisiana Standard** | **Explanations and Examples** |
| **A2: F-LE.A.2.** Given a graph, a description of a relationship, or two input-output pairs (include reading these from a table), construct linear and exponential functions, including arithmetic and geometric sequences to solve multi-step problems.  | Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions.**Examples**:* Determine an exponential function of the form *f*(*x*) *= abx* using data points from the table. Graph the function and identify the key characteristics of the graph.

|  |  |
| --- | --- |
| *x* | *f(x)* |
| 0 | 1 |
| 1 | 3 |
| 3 | 27 |

* Sara’s starting salary is $32,500. Each year she receives a $700 raise. Write a sequence in explicit form to describe the situation.
* After a record setting winter storm, there are 10 inches of snow on the ground! Now that the sun is finally out, the snow is melting. At 7 am there were 10 inches and at 12 pm there were 6 inches of snow.
1. Construct a linear function rule to model the amount of snow.
2. Construct an exponential function rule to model the amount of snow.
3. Which model best describes the amount of snow? Provide reasoning for your choice.
 |

| **A2: F-LE.A.4.** For exponential models, express as a logarithm the solution to *abct* = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. **★** | Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to analyze exponential models and evaluate logarithms.**Example**:* Solve 200 *e*0.04*t* = 450 for *t*.

*Solution:* We first isolate the exponential part by dividing both sides of the equation by 200.*e*0.04*t* = 2.25Now we take the natural logarithm of both sides.*ln* *e*0.04*t* = *ln* 2.25The left hand side simplifies to 0.04*t*, by logarithmic identity 1.0.04*t* = *ln* 2.25Lastly, divide both sides by 0.04.*t* = *ln* (2.25) / 0.04*t*  20.3 |
| --- | --- |

| Functions: Linear, Quadratic, and Exponential Models (F-LE)**Interpret expressions for functions in terms of the situation they model.** |
| --- |
| **Louisiana Standard** | **Explanations and Examples** |
| **A2: F-LE.B.5.** Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.**★** | Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions.**Examples:*** A function of the form *f*(*n*) = *P*(1 + *r*)*n* is used to model the amount of money in a savings account that earns 5% interest, compounded annually, where *n* is the number of years since the initial deposit. What is the value of *r*? What is the meaning of the constant *P* in terms of the savings account? Explain either orally or in written format.
* Lauren keeps records of the distances she travels in a taxi and what it costs:

|  |  |
| --- | --- |
| **Distance *d* in miles** | **Fare *f* in dollars** |
| 3 | 8.25 |
| 5 | 12.75 |
| 11 | 26.25 |

 1. If you graph the ordered pairs from the table, they lie on a line. How can this be determined without graphing them?
2. Show that the linear function in part a. has equation .
3. What do the 2.25 and the 1.5 in the equation represent in terms of taxi rides.
 |

| Functions: Trigonometric Functions (F-TF)**Extend the domain of trigonometric functions using the unit circle.** |
| --- |
| **Louisiana Standard** | **Explanations and Examples** |
| **A2: F-TF.A.1.** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. | Students know that if the length of an arc subtended by an angle is the same length as the radius of the circle, then the measure of the angle is 1 radian. Students should also determine the radian measures of angles subtended around the circle.**Examples:*** What is the radian measure of the angle t in the diagram ?

* The minute hand on the clock at the City Hall clock in Stratford measures 2.2 meters from the tip to the axle.
1. Through what angle does the minute hand pass between 7:07 a.m. and 7:43 a.m.?
2. What distance does the tip of the minute hand travel during this period?
 |
| **A2: F-TF.A.2.** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. | Students understand that one complete rotation around the unit circle, starting at (0,1), restricts the domain of trigonometric functions to . As more rotations are considered, the domain extends to all real numbers since the radian measure of any angle is a real number and there is no limit to the number of times one can travel around the unit circle. **Examples:*** Explain why and . Do these equations hold for any angle *θ*? Explain.
* Explain why and . Do these equations hold for any angle *θ*? Explain.
 |

| Functions: Trigonometric Functions (F-TF)**Model periodic phenomena with trigonometric functions.** |
| --- |
| **Louisiana Standard** | **Explanations and Examples** |
| **A2: F-TF.B.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **★** | Students choose the most appropriate trigonometric function to model a given situation dependent on the height and speed of the situation occurring. **Example:*** The temperature of a chemical reaction oscillates between a low of C and a high of C. The temperature is at its lowest point when *t* = 0 and completes one cycle over a six hour period.
1. Sketch the temperature, *T*, against the elapsed time, *t*, over a 12 hour period.
2. Find the period, amplitude, and the midline of the graph you drew in part a).
3. Write a function to represent the relationship between time and temperature.
4. What will the temperature of the reaction be 14 hours after it began?
5. At what point during a 24 hour day will the reaction have a temperature of C?
 |
| Functions: Trigonometric Functions (F-TF)**Prove and apply trigonometric identities.** |
| **Louisiana Standard** | **Explanations and Examples** |
| **A2: F-TF.C.8.** Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. | Students prove .In the unit circle, the cosine is the *x*-value, while the sine is the *y*-value. Since the hypotenuse is always 1, the Pythagorean relationship is always true. **Students use to find , , or given , , or and the quadrant of the angle.****Example:** Given and find and . |

| Statistics and Probability: Interpreting Categorical and Quantitative Data ★(S-ID)**Summarize, represent, and interpret data on a single count or measurement variable.** |
| --- |
| **Louisiana Standard** | **Explanations and Examples** |
| **A2: S-ID.A.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. | Students use the normal distribution to make estimates of frequencies (which can be expressed as probabilities**).** They recognize that only some data are well described by a normal distribution. They use the 68-95-99.7 rule to estimate the percent of a normal population that falls within 1, 2, or 3 standard deviations of the mean.**Examples:*** The bar graph below gives the birth weight of a population of 100 chimpanzees. The line shows how the weights are normally distributed about the mean, 3250 grams. Estimate the percent of baby chimps weighing 3000-3999 grams.

* Scores on a history test have a mean of 80 with standard deviation of 6. How many standard deviations from the mean is the student that scores a 90.
 |

| Statistics and Probability: Interpreting Categorical and Quantitative Data ★ (S-ID) **Summarize, represent, and interpret data on a two categorical and quantitative variables.** |
| --- |
| **Louisiana Standard** | **Explanations and Examples** |
| **A2: S-ID.B.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context*. Emphasize exponential models.*  | Write a function rule suggested by the context and determine how well it fits the data.**Examples:*** Investing Money: <https://www.khanacademy.org/math/probability/regression/prob-stats-scatter-plots/e/fitting-functions-to-scatter-plots>
* In 1985, there were 285 cell phone subscribers in the small town of Martinville. The table below shows the number of subscribers starting in 1986. Create a scatterplot and fit a function to the data. Approximately how many cell phone subscribers were in Martinville in 1994?

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Years | 1986 | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 |
| Number of Subscribers | 498 | 872 | 1527 | 2672 | 4677 | 8186 | 14325 | 25069 |  |

 |

| Statistics and Probability: Making Inferences and Justifying Conclusions ★ (S-IC) **Understand and evaluate random processes underlying statistical experiments.** |
| --- |
| **Louisiana Standard** | **Explanations and Examples** |
| **A2: S-IC.A.1.** Understand statistics as a process for making inferences about population parameters based on a random sample from that population. **★** | Student should be able to define populations, population parameter, random sample, and inference. * A *population* consists of everything or everyone being studied in an inference procedure. It is rare to be able to perform a census of every individual member of the population. Due to constraints of resources it is nearly impossible to perform a measurement on every subject in a population.
* A *parameter* is a value, usually unknown (and which therefore has to be estimated), used to represent a certain population characteristic.
* *Inferential statistics* considers a subset of the population. This subset is called a statistical sample often including members of a population selected in a random process. The measurements of the individuals in the sample tell us about corresponding measurements in the population.

Students demonstrate an understanding of the different kinds of sampling methods.**Example:** From a class containing 12 girls and 10 boys, three students are to be selected to serve on a school advisory panel. Here are four different methods of making the selection.1. Select the ﬁrst three names on the class roll.
2. Select the ﬁrst three students who volunteer.
3. Place the names of the 22 students in a hat, mix them thoroughly, and select three names from the mix.
4. Select the ﬁrst three students who show up for class tomorrow.

Which is the best sampling method, among these four, if you want the school panel to represent a fair and representative view of the opinions of your class? Explain the weaknesses of the three you did not select as the best. |
| **A2: S-IC.A.2.** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin will fall heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* **★** | Possible data-generating processes include (but are not limited to): flipping coins, spinning spinners, rolling a number cube, and simulations using the random number generators. Students may use graphing calculators, spreadsheet programs, or applets to conduct simulations and quickly perform large numbers of trials.The law of large numbers states that as the sample size increases, the experimental probability will approach the theoretical probability. Comparison of data from repetitions of the same experiment is part of the model building verification process. **Example:** * Have multiple groups flip coins. One group flips a coin 5 times, one group flips a coin 20 times, and one group flips a coin 100 times. Which group’s results will most likely approach the theoretical probability?
* Illustrative Mathematics – *Block Scheduling* at <http://www.illustrativemathematics.org/illustrations/125>
 |

| Statistics and Probability: Making Inferences and Justifying Conclusions ★ (S-IC)**Make inferences and justify conclusions from sample surveys, experiments, and observational studies.** |
| --- |
| **Louisiana Standard** | **Explanations and Examples** |
| **A2: S-IC.B.3.** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.**★** | Students understand the different methods of data collection, specifically the difference between an observational study and a controlled experiment, and know the appropriate use for each.* *Observational* *study* – a researcher collects information about a population by measuring a variable of interest, but does not impose a treatment on the subjects. (I.e. examining the health effects of smoking)
* *Experiment* – an investigator imposes a change or treatments on one or more group(s), often called treatment group(s). A comparative experiment is where a control group is given a placebo to compare the reaction(s) between the treatment group(s) and the control group.

Students understand the role that randomization plays in eliminating bias from collected data**.****Example:** Students in a high school mathematics class decided that their term project would be a study of the strictness of the parents or guardians of students in the school. Their goal was to estimate the proportion of students in the school who thought of their parents or guardians as “strict”. They do not have time to interview all 1000 students in the school, so they plan to obtain data from a sample of students.1. Describe the parameter of interest and a statistic the students could use to estimate the parameter.
2. Is the best design for this study a sample survey, an experiment, or an observational study? Explain your reasoning.
3. The students quickly realized that, as there is no definition of “strict”, they could not simply ask a student, “Are your parents or guardians strict?” Write three questions that could provide objective data related to strictness.
4. Describe an appropriate method for obtaining a sample of 100 students, based on your answer in part (a) above.
 |
| **A2: S-IC.B.4.** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.**★** | Students estimate a sample mean or sample proportion given data from a sample survey. Estimate the population value.**Examples:** * The label on a Barnum’s Animal Cracker box claims that there are 2 servings per box and a serving size is 8 crackers. The graph displays the number of animal crackers found in a sample of 28 boxes. Use the data from the 28 samples to estimate the average number of crackers in a box with a margin of error. Explain your reasoning or show your work.
* Margin of Error for Estimating a Population Mean (<https://www.illustrativemathematics.org/content-standards/HSS/IC/B/4/tasks/1956>)
 |
| **A2: S-IC.B.5.** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. **★** | Contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.**Examples:** * Sal purchased two types of plant fertilizer and conducted an experiment to see which fertilizer would be best to use in his greenhouse. He planted 20 seedlings and used Fertilizer A on ten of them and Fertilizer B on the other ten. He measured the height of each plant after two weeks. Use the data below to determine which fertilizer Sal should use.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Fertlizer A | 23.4 | 30.1 | 28.5 | 26.3 | 32.0 | 29.6 | 26.8 | 25.2 | 27.5 | 30.8 |
| Fertlizer B | 19.8 | 25.7 | 29.0 | 23.2 | 27.8 | 31.1 | 26.5 | 24.7 | 21.3 | 25.6 |

* 1. Use the data to generate simulated treatment results by randomly selecting ten plant heights from the twenty plant heights listed.
	2. Calculate the average plant height for each treatment of ten plants.
	3. Find the difference between consecutive pairs of treatment averages and compare. Does your simulated data provide evidence that the average plant heights using Fertlizer A and Fertilizer B is significant?
* “Are Starbucks customers more likely to be female?” To answer the question, students decide to randomly select 30-minute increments of time throughout the week and have an observer record the gender of every tenth customer who enters the Starbucks store. At the end of the week, they had collected data on 260 customers, 154 females and 106 males. This data seems to suggest more females visited Starbucks during this time than males.

To determine if these results are statistically significant, students investigated if they could get this proportion of females just by chance if the population of customers is truly 50% females and 50% males. Students simulated samples of 260 customers that are 50-50 females to males by flipping a coin 260 then recording the proportion of heads to represent the number of women in a random sample of 260 customers (e.g., 0.50 means that 130 of the 260 flips were heads). Their results are displayed in the graph at the right.Use the distribution to determine if the class’s data is statistically significant enough to conclude that Starbucks customers are more likely to be female. |
| **A2: S-IC.B.6.** Evaluate reports based on data.**★** | Contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.**Example:**Read the article below from NPR.org then answer the following questions.Kids and Screen Time: What Does the Research Say?By Juana SummersAugust 28, 2014Kids are spending more time than ever in front of screens, and it may be inhibiting their ability to recognize emotions, according to [new research out of the University of California, Los Angeles](http://newsroom.ucla.edu/releases/in-our-digital-world-are-young-people-losing-the-ability-to-read-emotions).[The study](http://www.sciencedirect.com/science/article/pii/S0747563214003227), published in the journal *Computers in Human Behavior*, found that sixth-graders who went five days without exposure to technology were significantly better at reading human emotions than kids who had regular access to phones, televisions and computers.The UCLA researchers studied two groups of sixth-graders from a Southern California public school. One group was sent to the [Pali Institute](http://www.paliinstitute.com/), an outdoor education camp in Running Springs, Calif., where the kids had no access to electronic devices. For the other group, it was life as usual.At the beginning and end of the five-day study period, both groups of kids were shown images of nearly 50 faces and asked to identify the feelings being modeled. Researchers found that the students who went to camp scored significantly higher when it came to reading facial emotions or other nonverbal cues than the students who continued to have access to their media devices."We were pleased to get an effect after five days," says Patricia Greenfield, a senior author of the study and a distinguished professor of psychology at UCLA. "We found that the kids who had been to camp without any screens but with lots of those opportunities and necessities for interacting with other people in person improved significantly more."If the study were to be expanded, Greenfield says, she'd like to test the students at camp a third time — when they've been back at home with smartphones and tablets in their hands for five days."It might mean they would lose those skills if they weren't maintaining continual face-to-face interaction," she says.1. Was this an experiment or an observational study?
2. What can you conclude?
3. Are there any limitations or concerns with this statistical study?
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| **Date Edited** | **STandard Code** | **type of edit made** |
| 06-15-2016 | A2: A-APR.D.6 | Removed an example that required finding the asymptote for a rational function. |
| 06-20-2016 | A2: SSE.B.4 | Edited text of standard to match that of crosswalk. |
| 09-21-2016 | A2: S-IC.A.1 | Edited text of standard to correct typing error. |

**UpDATES: Algebra II Companion document for TeacherS**